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# Complexity of Local F-Continuous Mapping Forms in Topological Groups with Fuzzy Singleton

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***Abstract***
*This research paper's main goal is to provide and examine specific generalized forms of continuous functions inside fuzzy topological groups. It clarifies fuzzy open homomorphisms as well.*
***Keywords:*** *Fuzzy topological group, Fuzzy open homomorphism, Fuzzy continuous maps, Fuzzy Singleton etcetera.*

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## I. Introduction

Fuzzy sets, which deal with reasoning and systems where the concepts of membership or belonging are not binary (i.e., in or out, true or false), were initially introduced by Zadeh [10]. Fuzzy mathematics allows for several levels of membership or belonging, represented by numbers in a range of the closed interval [0,1], as opposed to a rigid dichotomy: yes/no or true/false. As demonstrated by numerous writers who have examined the application of fuzzy sets in various domains, including medical research [7], safety and reliability engineering [6], and artificial intelligence [9], this improves human ability to deal with ambiguity and vagueness, which arise in many real-life circumstances. Several writers have proposed generalisations of fuzzy sets that consider the grade of non-membership under certain restrictions that preserve the total or power sum of both grades of membership and non-membership less or equal to one in order to tackle complex problems. These include intuitionistic fuzzy sets [5], Pythagorean fuzzy sets [8], (2,1)-fuzzy sets [2], $(m,n)$-fuzzy sets [3], soft $(m,n)$-fuzzy sets [4], complex nth power root fuzzy sets [1], and so on.

For all $h \in H$, the constant fuzzy sets with membership values of 0 and 1 are represented by $0_H$ and $1_H$, respectively. $J^H$, where $J = [0,1]$, is a collection of all fuzzy subsets in $H$. The negation of $K = 1_H - K$ defines the fuzzy complement of $K \in J^H$.

**Definition 1.1** Let $H$ be any group. Then, a fuzzy hausdorff space $(H,\epsilon)$ is said to be a strong fuzzy topological group if $R:\rho(H)\times\rho(H)\to\rho(H)$ defined by $R(s_x,t_y)=s_xt_y$ for every $(s_x,t_y)\in\rho(H)\times\rho(H)$ is continuous where $s_xt_y(uv)=\left(\mu_{s_xt_y}(uv),1-\mu_{s_xt_y}(uv)\right)=\left(\min\left(\mu_{s_x}(u),\mu_{t_y}(v)\right),\max\left(1-\mu_{s_x}(u),1-\mu_{t_y}(v)\right)\right)$.

**Definition 1.2** Let $H$ be any group. Then, a fuzzy hausdorff space $(H,\epsilon)$ is said to be a strong fuzzy topological group if $g:\rho(H)\to\rho(H)$ defined by $g(s_x)={s_x}^{-1}$, for every $s_x\in\rho(H)$ is continuous.

**Definition 1.3** Let $g:\rho(H)\to\rho(H)$ be the identity map, which is continuous then $u^{-1}=u\ \forall\ u\in\rho(H)$.

**Definition 1.4** If $R:\rho(H)\times\rho(H)\to\rho(H)$ defined by $R(s_x,t_y)=s_xt_y$ for every $(s_x,t_y)\in\rho(H)\times\rho(H)$ is fuzzy continuous, then $n:H\times H\to H$ defined by $n(u,v)=uv$ is fuzzy continuous.

**Definition 1.5** If $g:(H_1,\epsilon_1)\to(H_2,\epsilon_2)$ be a fuzzy continuous function between two fuzzy topological spaces $(H_1,\epsilon_1)$ and $(H_2,\epsilon_2)$ if and only if the induced function $R_f:\left(\rho(H_1),\tau_{\epsilon_1}\right)\to\left(\rho(H_2),\tau_{\epsilon_2}\right)$ is continuous.

**Definition 1.6** If $p_1,p_2\in\rho(H)$ and $t_1=\sup p_1$, $t_2=\sup p_2$. Then, $\mu_{p_1p_2}(t_1,t_2)\le\min\left(\mu_{p_1}(t_1),\mu_{p_2}(t_2)\right)\le\mu_{p_1'}(t_1)$ and $1-\mu_{p_1p_2}(t_1,t_2)\ge\max\left(1-\mu_{p_1}(t_1),1-\mu_{p_2}(t_2)\right)\ge1-\mu_{p_1'}(t_1)$, where $p_1p_2=\left(\mu_{p_1p_2},1-\mu_{p_1p_2}\right)$ is a fuzzy singleton defined on $t_1t_2$.

**Preposition 1.1** Let $(r_1,r_2)\in\rho(I)$ i.e. $r_1=\left(\mu_{r_1},1-\mu_{r_1}\right)\in\rho(I)$, $r_1=\left(\mu_{r_1},1-\mu_{r_1}\right)\in\rho(I)$ be fuzzy singleton defined on $u_1,u_2$ such that $r_1\in\partial_{\beta_1|I}$, $r_2\in\partial_{\beta_2|I}$, and $I$ is a subgroup of $H$. Then,$R_I\left(\partial_{\beta_1|I}\times\partial_{\beta_2|I}\right)\subseteq\partial_{\beta|I}$.

**2. New Results**

**Claim 2.1** For each $(s_u,t_v)\in\rho(H)\times\rho(H)$, if $R:\rho(H)\times\rho(H)\to\rho(H)$ defined by $R(s_u,t_v)=s_ut_v$ is continuous, then $n:H\times H\to H$ defined by $n(u,v)=uv$ is fuzzy continuous.

**Proof.** Assume that $R$ is continuous. Let $r=(\mu_r,1-\mu_r)$ be a fuzzy point in $H\times H$ with support $\{(u,v)\}$ in order to demonstrate that $n$ is a fuzzy continuous. Let $\mu_r(u,v)\in(0,1)$ and $1-\mu_r(u,v)\in(0,1)$. Let $H$ be a fuzzy open set with $\alpha = (\mu_\alpha, 1-\mu_\alpha)$ and $n(r) = \left(\mu_{n(r)}, 1-\mu_{n(r)}\right)$. The formula for $\mu_{n(r)}(x)$ is $\sup_{x_1x_2=x} (x_1, x_2)$. Thus, $n(r)$ is a fuzzy point with value $r(u,v)$ specified on $uv$. Define fuzzy singletons $r_1 = \left(\mu_{r_1}, 1-\mu_{r_1}\right)$ and $r_2 = \left(\mu_{r_2}, 1-\mu_{r_2}\right)$ on $u$ and $v$, respectively. $\mu_\alpha(uv)$ and $1-\mu_\alpha(uv)$ is the fuzzy value of a fuzzy singleton $\left(\mu_{r_1}, 1-\mu_{r_1}\right)$. The fuzzy values of a fuzzy singleton $\left(\mu_{r_1}, 1-\mu_{r_1}\right)$ are 1 and 0.

Since, $r_1r_2(uv) = \left(\mu_{r_1r_2}(uv), 1-\mu_{r_1r_2}(uv)\right) = \left(\min\left(\mu_{r_1}(u), \mu_{r_2}(v)\right), \max\left(1-\mu_{r_1}(u), 1-\mu_{r_2}(v)\right)\right) = \left(\mu_A(uv), 1-\mu_A(uv)\right)$ $r_1r_2 \in \alpha$ and $r_1r_2 \in \partial_\alpha$. There exist $\alpha_1 = \left(\mu_{\alpha_1}, 1-\mu_{\alpha_1}\right) \in \tau_\delta$ and $\alpha_2 = \left(\mu_{\alpha_2}, 1-\mu_{\alpha_2}\right) \in \tau_\delta$ such that $r_1 = \partial_{\alpha_1}, r_2 = \partial_{\alpha_2}$ and $\partial_{\alpha_1}\partial_{\alpha_2} \subseteq \partial_\alpha$ since $R$ is continuous and $r_1r_2 \in \partial_\alpha \in \tau_\delta$. We must demonstrate that $r \in \alpha_1 \times \alpha_2$, or $\mu_r \in \mu_{\alpha_1\times\alpha_2}$. In this case,$\mu_r(uv) = \mu_{n(r)}(uv) < \mu_\alpha(uv) = \min\left(\mu_{r_1}(u), \mu_{r_2}(v)\right) \le \min\left(\mu_{\alpha_1}(u), \mu_{\alpha_2}(v)\right) = \mu_{\alpha_1\times\alpha_2}$
$(u,v)$so $r \in \alpha_1 \times \alpha_2$. We now demonstrate the idea that $n(\alpha_1 \times \alpha_2)$ is a subset of $\alpha$. By definition, $\mu_{n(\alpha_1\times\alpha_2)}(x) = \sup_{x_1x_2=x} \min\left(\mu_{\alpha_1}(x_1), \mu_{\alpha_2}(x_2)\right) = \mu_{\alpha_1\times\alpha_2}(x_1, x_2)$.

Now, $\partial_{\alpha_1}\partial_{\alpha_2} \subseteq \partial_\alpha$ i.e. $\min\left(\mu_{\alpha_1}(u), \mu_{\alpha_2}(v)\right) = \mu_\alpha(uv)$ then as a result, $\mu_{(\alpha_1\times\alpha_2)}(x_1, x_2) \le \mu_\alpha(x_1, x_2) = \mu_\alpha(x)$, $\forall\, u, v \in H$. Therefore, $\mu_{n(\alpha_1\times\alpha_2)}(x) \le \mu_\alpha(x)$. Therefore, $n$ is fuzzy continuous according to the notion of fuzzy continuity.

**Claim 2.2** If $(H, \epsilon)$ is a fuzzy topological space on group $G$, then $(H, \epsilon)$ is a strong fuzzy topological group if and only if $R_f\colon \rho(H) \times \rho(H)$ defined by $R_f(s_x, t_y) = s_x{t_y}^{-1}$, $\forall\, (s_x, t_y) \in \rho(H) \times \rho(H)$ is continuous.

**Proof.** Assume that the fuzzy topological group $(H, \epsilon)$ is strong. Let $t_y = \mu_{t_y}$ and $s_x = \mu_{s_x}$ be fuzzy singletons defined on $x$ and $y$ respectively. Let $g_1, g_2\colon \rho(H) \to \rho(H)$ be defined by $g_1(s_x) = s_x$ and $g_2(t_y) = {t_y}^{-1}$ respectively. It is evident that $g_1$ and $g_2$ are continuous. Thus, $g(s_x, t_y) = \left(s_x{t_y}^{-1}\right)$ defines the function $g\colon \rho(H) \times \rho(H) \to \rho(H) \times \rho(H)$. $\forall\, (s_x, t_y) \in \rho(H) \times \rho(H)$ is continuous. Since $R_f(s_x, t_y) = s_x{t_y}^{-1} = R\left(g(s_x, t_y)\right) = (R.g)(s_x, t_y)$, $R_f$ is continuous.
On the other hand, suppose that $R_f(s_x, t_y) = s_x{t_y}^{-1}\forall\, (s_x, t_y) \in \rho(H) \times \rho(H)$ is continuous. Naturally, $f(s_x) = {s_x}^{-1} = R_f(h(s_x)) = (R_f.g)(s_x)$ is continuous since the function $h\colon \rho(H) \to \rho(H) \times \rho(H)$ defined by $h(s_x) = (f_e, s_x)$ is continuous. $g_1\colon \rho(H) \to \rho(H)$ is $g_1(s_x) = s_x$, then $\tau\colon \rho(H) \times \rho(H) \to \rho(H) \times \rho(H)$ is continuous if $\tau(s_x, t_y) = (s_x, f)(s_x, t_y) = \left(s_x{t_y}^{-1}\right)$.
Since $R(s_x, t_y) = R_f\left(\tau(s_x, t_y)\right) = R_f.\tau$, $R$ is continuous, and $(H, \epsilon)$ is a strong fuzzy topological group.

**Claim 2.3** Let $I$ be a fuzzy topological subgroup of $(H', \epsilon)$ and let $\varphi\colon (H, \epsilon) \to (H', \epsilon)$ be a continuous fuzzy open homomorphism. If $\varphi(\beta) \in \rho$, then $\beta = (\mu_\beta, 1-\mu_\beta) \in \theta$ and $\alpha = \beta|\varphi^{-1}(I)$, meaning that $\mu_\beta = \mu_\beta|\varphi^{-1}(I)$ and $1-\mu_\beta = 1-\mu_\beta|\varphi^{-1}(I)$. Furthermore, $R_{f_I}(\partial_{\beta_1} \times \partial_{\beta_2}) \subseteq \partial_{\varphi(\beta)|I}$ $\mu_{t_1't_2'^{-1}} \le \mu_{\varphi(\beta)|I}$, and $\mu_{\varphi(\beta)|I} - \mu_{t_1't_2'^{-1}} \le 0$. Next, $\mu_{t_1t_2^{-1}}(v_1v_2^{-1}) \le \mu_{\varphi(\beta)|I}(v_1v_2^{-1})$, $\mu_{\varphi(\beta)|I}(v_1v_2^{-1}) - \mu_{t_1t_2^{-1}}(v_1v_2^{-1}) \le 0$ and $\mu_\alpha(u_1u_2'^{-1}) \ge \mu_{s_1's_2'^{-1}}(u_1u_2'^{-1})$, $\mu_\alpha(u_1u_2'^{-1}) - \mu_{s_1's_2'^{-1}}(u_1u_2'^{-1}) \le 0$.

**Proof.** Given that $\varphi$ is a fuzzy homomorphism and $I$ is a subgroup, $\left(\mu_{t_1t_2^{-1}}(v_1v_2^{-1})\right) = \min\left(\mu_{t_1}(v_1), \mu_{t_2^{-1}}(v_2^{-1})\right) \le \min\left(\mu_{s_1}(u_1), \mu_{s_2^{-1}}(u_2^{-1})\right) = \mu_{s_1s_2^{-1}}(u_1u_2^{-1}) \le \mu_\alpha(u_1u_2^{-1}) = \mu_{\varphi^{-1}(\beta|I)}(u_1u_2^{-1}) \le \mu_{\varphi(\beta)|I}(v_1v_2^{-1})$ and similarly $\left(1-\mu_{t_1t_2^{-1}}(v_1v_2^{-1})\right) = \max\left(1-\mu_{t_1}(v_1), \mu_{t_2^{-1}}(v_2^{-1})\right) \ge \max\left(1-\mu_{s_1}(u_1), \mu_{s_2^{-1}}(u_2^{-1})\right) = 1-\mu_{s_1s_2^{-1}}(u_1u_2^{-1}) \ge \mu_\alpha(u_1u_2^{-1}) = 1-\mu_{\varphi^{-1}(\beta|I)}(u_1u_2^{-1}) \ge 1-\mu_{\varphi(\beta)|I}(v_1v_2^{-1})$, so the outcome is $\mu_\alpha(u_1u_2'^{-1}) - \mu_{s_1's_2'^{-1}}(u_1u_2'^{-1}) \le 0$. Once more, since $R_{f_I}(\partial_{\beta_1} \times \partial_{\beta_2}) \subseteq \partial_{\varphi(\beta)|I}$, $\mu_{t_1't_2'^{-1}} \le \mu_{\varphi(\beta)|I}$ and $\mu_{\varphi(\beta)|I} - \mu_{t_1't_2'^{-1}} \ge 0$. Then $\mu_{s_1's_2'^{-1}}(u_1u_2'^{-1}) = \mu_{t_1't_2'^{-1}}(v_1v_2'^{-1}) \le \mu_{\varphi(\beta)|I}(v_1v_2^{-1}) = \mu_\beta(u_1u_2'^{-1})$ $\mu_\alpha(u_1u_2'^{-1})$ and $1-\mu_{s_1's_2'^{-1}}(u_1u_2'^{-1}) = 1-\mu_{t_1't_2'^{-1}}(v_1v_2'^{-1}) \ge 1-\mu_{\varphi(\beta)|I}(v_1v_2^{-1}) = 1-\mu_\beta(\varphi(v_1)\varphi(v_2')^{-1}) = 1-\mu_\beta(u_1u_2'^{-1})$. So, the outcome is $\mu_\alpha(u_1u_2'^{-1}) - \mu_{s_1's_2'^{-1}}(u_1u_2'^{-1}) \ge 0$. Hence, the result.

**Claim 2.4** If $\varphi\colon \rho(H_1 \times H_2) \times \rho(H_1 \times H_2) \to \rho(H_1 \times H_2)$ defined by $\varphi(r_u, s_v) = r_us_v$, where $r_u$ and $s_v$ are fuzzy singletons defined on $(u_1, u_2)$ and $(v_1, v_2)$ respectively. Then, $\varphi(\partial_\beta \times \partial_\gamma) \subseteq \partial_\alpha$. Also, if $g\colon (H_1, \epsilon_1) \to (H_2, \epsilon_2)$ be fuzzy open, $g(\alpha_1), g(\alpha_2) \in \epsilon_2$ with $g(\alpha_1)|g(I) = g(\beta_1)$, $g(\alpha_2)|g(I) = g(\beta_2)$ and $s_1 \in \beta_1$. Then $\mu_{t_1}(v_1) \le \mu_{g(\beta_1)}(v_1)$ and $\mu_{g(\beta_1)}(v_1) - \mu_{t_1}(v_1) \ge 0$.

**Proof.** Let $\varepsilon \in \partial_\beta$ and $\epsilon \in \partial_\gamma$ where $\varepsilon$ and $\epsilon$ are fuzzy singleton defined on $(x_1, x_2)$ and $(y_1, y_2)$ respectively. Now, we have to prove that

$\varepsilon\epsilon \in \partial_{\alpha}.\mu_{\varepsilon\epsilon}(x_1 *_1 y_1, x_2 *_2 y_2) = \min(\mu_{\varepsilon}(x_1, x_2), \mu_{\epsilon}(y_1, y_2)) \leq \min\left(\mu_{\beta}(x_1, x_2), \mu_{\gamma}(y_1, y_2)\right) = \min\left((\mu_{\beta_1} \times \mu_{\beta_2})(x_1 x_2), (\mu_{\gamma_1} \times \mu_{\gamma_2})(y_1, y_2)\right) = \min\left((\mu_{\beta_1} \times \mu_{\gamma_1})(x_1 y_1), (\mu_{\beta_2}\mu_{\gamma_2})(x_2, y_2)\right) \leq \min(\mu_{\alpha_1}(x_1 *_1 y_1), \mu_{\alpha_2}(x_2 *_2 y_2)) = (\mu_{\alpha_1} \times \mu_{\alpha_2})(x_1 *_1 y_1, x_2 *_2 y_2)$ and

$1 - \mu_{\varepsilon\epsilon}(x_1 *_1 y_1, x_2 *_2 y_2) = \max(1 - \mu_{\varepsilon}(x_1, x_2), 1 - \mu_{\epsilon}(y_1, y_2)) \geq \max\left(1 - \mu_{\beta}(x_1, x_2), 1 - \mu_{\gamma}(y_1, y_2)\right) = \max\left(\left((1 - \mu_{\beta_1}) \times (1 - \mu_{\beta_2})\right)(x_1 x_2), \left((1 - \mu_{\gamma_1}) \times (1 - \mu_{\gamma_2})\right)(y_1, y_2)\right) = \max\left(\left((1 - \mu_{\beta_1}) \times (1 - \mu_{\gamma_1})\right)(x_1 y_1), \left((1 - \mu_{\beta_2}) \times (1 - \mu_{\gamma_2})\right)(x_2, y_2)\right) \geq \max\left((1 - \mu_{\alpha_1})(x_1 *_1 y_1), (1 - \mu_{\alpha_2})(x_2 *_2 y_2)\right) = \left((1 - \mu_{\alpha_1}) \times (1 - \mu_{\alpha_2})\right)(x_1 *_1 y_1, x_2 *_2 y_2)$.

Hence $\varepsilon\epsilon \in \partial_{\beta_1 \times \beta_2} \subseteq \partial_{\alpha}$ and on the other hand $s_1 \in \beta_1$, $\mu_{s_1} \leq \mu_{\beta_1} = \mu_{\alpha|I}$ and $1 - \mu_{s_1} \leq 1 - \mu_{\beta_1} = 1 - \mu_{\alpha|I}$. Now, $\mu_{t_1}(v_1) = \mu_{s_1}(u_1) \leq \mu_{\beta_1}(u_1) = \mu_{g(\beta_1)}(v_1)$ and $1 - \mu_{t_1}(v_1) = 1 - \mu_{s_1}(u_1) \geq 1 - \mu_{\beta_1}(u_1)$. Thus, $\mu_{g(\beta_1)}(v_1) - \mu_{t_1}(v_1) \geq 0$. Ofcourse, $(t_1, t_2) \in \partial_{g(\beta_1)} \times \partial_{g(\beta_2)}$ and hence, the result.

**Final remarks**

This study examines the features of continuous functions in simple fuzzy topological groups and validates previous claims about them. This idea can be expanded in the future to include Pythagorean fuzzy sets, rough fuzzy sets, and intuitionistic fuzzy settings. As a result, this provides a fresh perspective on fuzzy topological algebraic structures.

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