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Spanning Cactus in Desargues Graph *

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Abstract. *Spanning Cactus Existence Problem is NP-Complete for general Graphs. Here, the spanning cactus existence problem is studied for the Desargues graph. Desargues graph is the Generalized Petersen Graph G(10, 3). Here we have shown that there exists a spanning cactus in the Desargues graph. Further, it is shown that there exists only one spanning cactus which is a Hamiltonian cycle of the graph.*
Key words. *Desargues graph, Cactus, Spanning cactus, SPEC, Generalized Petersen Graph G(10, 3), Cubic graph*

I. Introduction.

Let $G(V, E)$ be a connected graph. If each of the edges in G is contained in exactly one cycle, then the graph $G(V, E)$ is called a Cactus. Cactus graphs are well-studied graphs and have many applications such as in Networking [1][2][5], Genome comparison [18], etc. We say a graph G has a spanning cactus if there is a subgraph in G which is a cactus, spanning all the vertices of G. The Spanning cactus existence problem is the problem of testing whether G has a spanning cactus or not. The areas of applications of the problem also include the field of Networking, Electrical circuits, genome comparisons, etc. The Spanning Cactus Existence Problem is NP-complete in general graphs[15]. Palbom has shown that the directed spanning cactus prob- lem for general di-graphs is also NP-Complete[17]. There is a 2-approximation algorithm for the Minimum Cactus Extension Problem (MCEP) to extend a spanning forest to a spanning cactus when the edge costs satisfy the triangle inequality [7]. There is a τ -approximation algorithm for the minimum span- ning cactus problem in general graphs [6]. The existence of a spanning Cactus is also studied for many special graphs. In [9], the authors have shown that there exists a spanning cactus in the $3 \times 3 \times 3$ Grid graph. Whereas in [8], it is shown that there does not exist any spanning Cactus for Petersen Graph. Here, I have studied the spanning cactus existence problem in the Desargues graph. It is shown that there exists a spanning cactus in the Desargues graph. Further, I have shown that this spanning cactus is unique. The result is use- ful as the graph has many applications in the areas such as Network circuits, Chemical Databases, etc. We have shown that there exists a spanning cactusin the Desargues graph.

The Desergues graph and its properties are presented in section 2. In sec- tion 3, I have shown that the Desergues graph has a spanning cactus. Further, I have shown that the spanning cactus is unique. The concluding remarks are presented in section 4.

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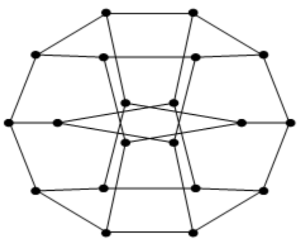
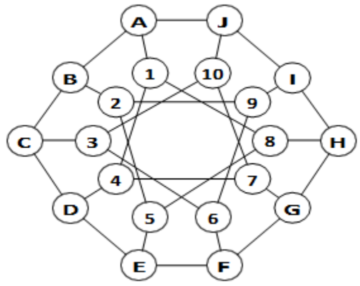
II. Preliminaries.

A cubic symmetric graph with 20 vertices and 30 edges is called a Desargues graph [19]. Figure 1 presents a Desargues graph. The Desargues graph is distance-transitive and non-planar [14]. Bussemaker et. al. have shown that there are exactly 13 connected, cubic integral graphs and Desargues graph is one of them [4]. Desargues graph is 3-unitransitive [13] and a unit-distance graph [12] [19] . It is also isomorphic to Generalized Petersen Graph *GP* (10, 3) [19][13]. The Generalized Petersen Graph *GP* (10, 3) can be formed by connecting the vertices of a regular decagon to the corresponding vertices of a ten-pointed star polygon [16]. More precisely, there are 10 vertices that form a regular decagon, and the other 10 vertices form another decagon that connects pairs of vertices at distance three which is a ten-pointed star. There are edges with a vertex in one decagon to a corresponding vertex in the other, resulting in total of 30 edges in the graph as in Figure 1b.



(a) A Desargues Graph



(b) Another representation of Desargues Graph

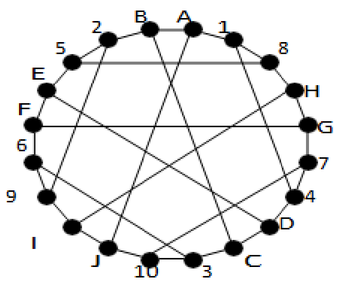
Fig. 1: Desargues Graphs

III. Proof of existence.

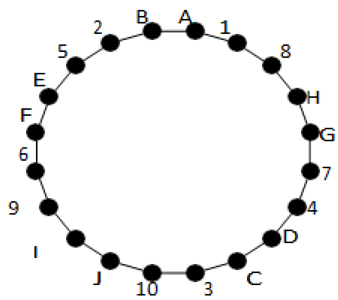
A Hamiltonian Cycle in a graph $G(V, E)$ is a simple cycle that visits every vertex $v \in V(G)$ exactly once except the start vertex which is visited twice. A graph is called Hamiltonian if there exists a Hamiltonian cycle in it. The Hamiltonian Cycle problem is a well-known classical problem in Graph Theory. The problem is NP-complete for general Graph [10] [11]. In [3], the author has proved that the Generalized Petersen Graph (*GP* (10, 3)) is Hamiltonian Lemma 1. Consider the graph in Figure 1b. The sequence of vertices {*A, B,* 2, 5, *E, F,* 6, 9, *I, J,* 10, 3, *C, D,* 4, 7, *G, H,* 8, 1, *A*} is a Hamilto-nian cycle as shown in Figure 2a.

LEMMA 1. *[3] The Generalized Petersen Graph* (*GP*(10, 3)) *is Hamiltonian.*

The spanning cactus existence problem is related to the Hamiltonian cycle problem in the sense that the existence of the Hamiltonian cycle also proves the existence of a spanning cactus as the generated cycle will be a spanning



(a) Hamiltonian Cycle shown



(b) The Spanning Cactus

Fig. 2: A Hamiltonian Cycle and the Spanning Cactus in the Desargues Graph with vertex labeling as shownin Figure 1b

cactus in the given graph.

THEOREM 2. *Spanning cactus exists in the Desargues Graph.*

Proof. Since the existence of a Hamiltonian cycle also proves the existence of Spanning cactus, there exists a spanning cactus in the Generalized Petersen Graph *GP* (10, 3) by Lemma 1[3]. Since the Desargues Graph is isomorphic to the Generalized Petersen Graph *GP* (10, 3), a spanning cactus exists in it. In Figure 2b, a Spanning cactus is shown corresponding to the Desargues Graph with vertex labeling as shown in Figure 1b. And the subgraph induced by the edges in the Hamiltonian Cycle is the resultant spanning cactus as shown in Figure 2b.

THEOREM 3. *There exists a unique spanning cactus in the Desargues Graph.*

Proof. If possible, let *C* be a spanning cactus of the Desargues graph *G* which is not a Hamiltonian cycle. Then *C* must have a vertex *v* of degree 4. Since degree of the vertex *v* in *G* is exactly 3, *v* can not have degree 4 in *C*, a contradiction.

IV. Conclusion.

I have shown that spanning cactus exists in the Desargues Graph. Further, I have shown that it is a Hamiltonian cycle and this span- ning cactus is unique.

REFERENCES

[1] R. K. AHUJA, T. L. MAGNANTI, AND J. B. ORLIN, Network flows: Theory, algorithms, and applications, (1993).

[2] B. BEN-MOSHE, A. DVIR, M. SEGAL, AND A. TAMIR, Centdian computation in cactus graphs., J. Graph Algorithms Appl.,16 (2012), pp. 199–224.

[3] J. A. BONDY, Variations on the hamiltonian theme, Canadian Mathematical Bulletin, 15 (1972), p. 57–62, doi:10.4153/CMB-1972-012-3.

[4] F. BUSSEMAKER AND D. CVETKOVIĆ, There are exactly 13 connected, cubic, integral graphs, Publikacije Elektrotehničkog fakulteta. Serija Matematika i fizika, (1976), pp. 43–48.

[5] K. DAS AND M. PAL, An optimal algorithm to find maximum and minimum height spanning trees on cactus graphs, AdvancedModeling and Optimization, 10 (2008), pp. 121–134.

[6] A. K. DATTA, Approximate spanning cactus, Information Processing Letters, 115 (2015), pp. 828–832, doi:https://doi.org/10.1016/j.ipl.2015.06.009, https://www.sciencedirect.com/science/article/pii/S0020019015001052.

[7] A. K. DATTA AND C. DEBNATH, Spanning cactus: Complexity and extensions, Discrete Applied Mathematics, 233 (2017), pp. 19–28.

[8] C. DEBNATH AND A. K. DATTA, A short note on spanning cactus problem of petersen graph, in Intelligent Techniques and Applications in Science and Technology, S. Dawn, V. E. Balas, A. Esposito, and S. Gope, eds., Cham, 2020, SpringerInternational Publishing, pp. 757–760.

[9] C. DEBNATH AND A. K. DATTA, Spanning cactus existence in a three-dimensional (3 x3 x3) grid, in Proceedings of the International Conference on Innovative Computing & Communications (ICICC), 2020.

[10] M. R. GAREY AND D. S. JOHNSON, Computers and Intractibility: A guide to the theory of NP-completeness, W.H. Freemanand Co.,San Francisco, 1979.

[11] M. R. GAREY, D. S. JOHNSON, AND R. E. TARJAN, The planar hamiltonian circuit problem is np-complete, SIAM J. Comput., 5 (1976), pp. 704–714.

[12] E. GERBRACHT, On the unit distance embeddability of connected cubic symmetric graphs, in Kolloquium über Kombinatorik. Magdeburg, Germany, 2008.

[13] F. HARARY, Graph theory, Addison-Wesley, (1969).

[14] Y. HU AND H. LUO, Study of the non-planarity of the desargues graph, in Advances in Computer Science, Intelligent System and Environment, D. Jin and S. Lin, eds., Berlin, Heidelberg, 2011, Springer Berlin Heidelberg, pp. 7–11.

[15] S. N. KABADI AND A. P. PUNNEN, Spanning cactus of a graph: Existence, extension, optimization and approximation, Discrete Applied Mathematics, 161 (2013), pp. 167–175, doi:https://doi.org/10.1016/j.dam.2012.08.006, https://www. sciencedirect.com/science/article/pii/S0166218X1200306X.

[16] M. KRNC AND T. PISANSKI, Characterization of generalized petersen graphs that are kronecker covers, arXiv preprint arXiv:1802.07134, (2018).

[17] A. PALBOM, Complexity of the directed spanning cactus problem, Discrete Applied Mathematics, 146 (2005), pp. 81–91, doi:https://doi.org/10.1016/j.dam.2004.08.006, https://www.sciencedirect.com/science/article/pii/S0166218X04002914.

[18] B. PATEN, M. DIEKHANS, D. EARL, J. S. JOHN, J. MA, B. SUH, AND D. HAUSSLER, Cactus graphs for genome comparisons, Journal of Computational Biology, 18 (2011), pp. 469–481.

[19] E. W. WEISSTEIN, Desargues graph, https://mathworld. wolfram. com/, (2002).