Fuzzy W- Super Continuous Mappings

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Abstract: The purpose of this paper to introduce and study the concepts of fuzzy w-super closed sets and fuzzy w-super continuous mappings in fuzzy topological spaces.

Keywords: fuzzy super closure fuzzy super interior, fuzzy super open set , fuzzy super closed set , fuzzy w-super closed set , fuzzy g-super closed set, fuzzy g- super open set, fuzzy sg-super closed set, fuzzy sg-super open set, fuzzy gs-super closed set.

I. Preliminaries

Let X be a non empty set and I= [0,1]. A fuzzy set on X is a mapping from X to I which assumes only the value is 0 and whole fuzzy sets 1 is a mapping from X to I which takes the values 1 only. The union (resp. intersection) of a family \( \{A_{\alpha}: \alpha \in \Lambda \} \) of fuzzy sets of X is defined by to be the mapping sup \( A_{\alpha} \) (resp. inf \( A_{\alpha} \)). A fuzzy set A of X is contained in a fuzzy set B of X if \( A(x) \leq B(x) \) for each \( x \in X \). A fuzzy point \( x_\beta \) in X is a fuzzy set defined by \( x_\beta(y) = \beta \) for \( y = x \) and \( x(y) = 0 \) for \( y \neq x \), \( \beta \in [0,1] \) and \( y \in X \). A fuzzy set \( x_\beta \) is said to be quasi-coincident with the fuzzy set A denoted by \( x_\betaqA \) if and only if \( \beta + A(x) > 1 \). A fuzzy set A is quasi-coincident with a fuzzy set B denoted by \( AqB \) if and only if there exists a point \( x \in X \) such that \( A(x) + B(x) > 1 \). A ≤ B if and only if \( A \subseteq B \).

A family \( \tau \) of fuzzy sets of X is called a fuzzy topology [2] on X if \( 0,1 \) belongs to \( \tau \) and \( \tau \) is super closed with respect to arbitrary union and finite intersection. The members of \( \tau \) are called fuzzy super open sets and their complement are fuzzy super closed sets. For any fuzzy set A of X the closure of A (denoted by cl(A)) is the intersection of all the fuzzy super closed sets of A and the interior of A (denoted by int(A)) is the union of all fuzzy super open subsets of A.

Definition 1.1 [5]: Let \( (X, \tau) \) fuzzy topological space and \( A \subseteq X \) then

1. Fuzzy Super closure \( scl(A) = \{x \in X : cl(U) \cap A \neq \emptyset \} \)
2. Fuzzy Super interior \( sint(A) = \{x \in X : cl(U) \subseteq A \neq \emptyset \} \)

Definition 1.2[5]: A fuzzy set A of a fuzzy topological space \( (X, \tau) \) is called:
(a) Fuzzy super closed if \( scl(A) \leq A \).
(b) Fuzzy super open if \( 1-A \) is fuzzy super closed \( sint(A) = A \)

Remark 1.1[5]: Every fuzzy closed set is fuzzy super closed but the converses may not be true.

Remark 1.2[5]: Let A and B are two fuzzy super closed sets in a fuzzy topological space \( (X, \tau) \), then \( A \cup B \) is fuzzy super closed.

Remark 1.3[5]: The intersection of two fuzzy super closed sets in a fuzzy topological space \( (X, \tau) \) may not be fuzzy super closed.

Definition 1.3[1,5,6,7]: A fuzzy set A of a fuzzy topological space \( (X, \tau) \) is called:
(a) fuzzy semi super open if there exists a super open set O such that \( O \subseteq A \subseteq cl(O) \).
(b) fuzzy semi super closed if its complement 1-A is fuzzy semi super open.
Remark 1.4[1,5,7]: Every fuzzy super open (resp. fuzzy super closed) set is fuzzy semi super open (resp. fuzzy semi super closed) but the converse may not be true.

Definition 1.4[5]: The intersection of all fuzzy super closed sets which contains A is called the semi super closure of a fuzzy set A of a fuzzy topological space (X, τ). It is denoted by scl(A).

Definition 1.5[3,11,8,9,10]: A fuzzy set A of a fuzzy topological space (X, τ) is called:
1. fuzzy g-super closed if cl(A) ≤ G whenever A ≤ G and G is super open.
2. fuzzy g-super open if its complement 1-A is fuzzy g-super closed.
3. fuzzy sg-super closed if scl(A) ≤ O whenever A ≤ O and O is fuzzy semi super open.
4. fuzzy sg-super open if its complement 1-A is sg-super closed.
5. fuzzy gs-super closed if scl(A) ≤ O whenever A ≤ O and O is fuzzy super open.
6. fuzzy gs-super open if its complement 1-A is gs-super closed.

Remark 1.5[10,11]: Every fuzzy super closed (resp. fuzzy super open) set is fuzzy g-super closed (resp. fuzzy g-super open) and every fuzzy g-super closed (resp. fuzzy g-super open) set is fuzzy gs-super closed (resp. gs-super open) but the converses may not be true.

Remark 1.6[10,11]: Every fuzzy semi super closed (resp. fuzzy semi super open) set is fuzzy sg-super closed (resp. fuzzy sg-super open) and every fuzzy sg-super closed (resp. fuzzy sg-super open) set is fuzzy gs-super closed (resp. gs-super open) but the converses may not be true.

Definition 1.6[10,11]: A mapping from a fuzzy topological space (X, τ) to a fuzzy topological space (Y, Γ) is said to be:
1. fuzzy super continuous if f\(^{-1}\)(G) is fuzzy super open set in X for every fuzzy super open set G in Y.
2. fuzzy semi super continuous if f\(^{-1}\)(G) is fuzzy semi super open set in X for every fuzzy super open set G in Y.
3. fuzzy irresolute if the inverse image of every fuzzy semi super open set of Y is fuzzy semi super open in X.
4. fuzzy g-super continuous if f\(^{-1}\)(G) is fuzzy g-super closed set in X for every fuzzy super closed set G in Y.
5. fuzzy sg-super continuous if f\(^{-1}\)(G) is fuzzy sg-super closed set in X for every fuzzy super closed set G in Y.
6. fuzzy gs-super continuous if f\(^{-1}\)(G) is fuzzy gs-super closed set in X for every fuzzy super closed set G in Y.

Remark 1.7[1, 11]: Every fuzzy super continuous mapping is fuzzy g-super continuous (resp. fuzzy semi super continuous) but the converse may not be true.

Remark 1.8[10, 11]: Every fuzzy g-super continuous mapping is fuzzy gs-super continuous but the converses may not be true.

Remark 1.9[10, 11]: Every fuzzy semi super continuous mapping is fuzzy sg-super continuous and every sg-super continuous mapping is gs-super continuous but the converses may not be true.

Definition 1.6[10, 11]: A fuzzy topological space (X, τ) is said to be fuzzy T\(_{1/2}\) (resp. fuzzy semi T\(_{1/2}\)) if every fuzzy g-super closed (resp. fuzzy sg-super closed) set in X is fuzzy super closed (resp. fuzzy semi super closed).

Definition 1.7: A fuzzy set A of a fuzzy topological space (X, τ) is called fuzzy w-super closed if cl(A) ≤ U whenever A ≤ U and U is fuzzy semi super open.

Remark 1.9: Every fuzzy super closed set is fuzzy w-super closed but its converse may not be true. For,

Example 2.1: Let X = \{a, b\} and the fuzzy sets A and U are defined as follows:
A(a)= 0.5, A(b)=0.5 U(a)= 0.5,U(b)=0.4
Let $\mathcal{T} = \{0, 1, U\}$ be a fuzzy topology on $X$. Then the fuzzy set $A$ is fuzzy $w$-super closed but it is not fuzzy super closed.

**Remark 1.10:** Every fuzzy $w$-super closed set is fuzzy $g$-super closed but the converse may not be true. For,

**Example 2.:** Let $X = \{a, b\}$ and the fuzzy sets $A$ and $U$ are defined as follows:

$U(a) = 0.7, U(b) = 0.6, A(a) = 0.6, A(b) = 0.7$

Let $\mathcal{T} = \{0, 1, U\}$ be a fuzzy topology on $X$. Then the fuzzy set $A$ is fuzzy $g$-super closed but it is not fuzzy $w$-super closed.

**Remark 1.11:** Every fuzzy $w$-super closed set is fuzzy $sg$-super closed, but the converse may not be true. For,

**Example 2.2:** Let $X = \{a, b\}$ and the fuzzy sets $A$ and $U$ be defined as follows:

$A(a) = 0.5, A(b) = 0.3, U(a) = 0.5, U(b) = 0.4$.

Let $\mathcal{T} = \{0, U, 1\}$ be a fuzzy topology on $X$. Then the fuzzy set $A$ is fuzzy $sg$-super closed but it is not fuzzy $w$-super closed.

**Remark 1.12:** If $A$ and $B$ are fuzzy $w$-super closed sets in a fuzzy topological space $(X, \tau)$ then $A \cup B$ is fuzzy $w$-super closed.

**Remark 1.13:** The intersection of any two fuzzy $w$-super closed sets in a fuzzy topological space $(X, \tau)$ may not be fuzzy $w$-super closed for,

**Example 2.3:** Let $X = \{a, b\}$ and the fuzzy sets $U$ and $A$ are defined as follows:

$U(a) = 1, U(b) = 0, A(a) = 1, A(b) = 0$.

Let $\mathcal{T} = \{0, U, 1\}$ be a fuzzy topology on $X$. Then $A$ and $B$ are fuzzy $w$-super closed set in $(X, \tau)$ but $A \cap B$ is not fuzzy $w$-super closed.

**Remark 1.14:** Let $A \leq B \leq \text{cl}(B)$ and $A$ is fuzzy $w$-super closed set in a fuzzy topological space $(X, \tau)$ then $B$ is fuzzy $w$-super closed.

**Definition 1.7:** A fuzzy set $A$ of a fuzzy topological space $(X, \tau)$ is called fuzzy $w$-super open if and only if $1 - A$ is fuzzy $w$-super closed.

**Remark 1.15:** Every fuzzy super open (resp. fuzzy $g$-super open) set is fuzzy $w$-super open. But the converse may not be true. For the fuzzy set $B$ defined by $B(a) = 0.5, B(b) = 0.5$ in the fuzzy topological space $(X, \tau)$ of example (2.1) is fuzzy $w$-super open but it is not fuzzy super open and the fuzzy set $C$ defined by $C(a) = 0.4, C(b) = 0.3$ in the fuzzy topological space $(X, \tau)$ of example (2.2) is fuzzy $w$-super open but it is not fuzzy $g$-super open.

**Remark 1.16:** A fuzzy set $A$ of a fuzzy topological space $(X, \tau)$ is fuzzy $w$-super open if and only if $F \leq \text{int}(A)$ whenever $F \leq A$ and $F$ is fuzzy semi super closed.

**Remark 1.17:** Let $A$ be a fuzzy $w$-super open subset of a fuzzy topological space $(X, \tau)$ and $\text{int}(A) \leq B \leq A$ then $B$ is fuzzy $w$-super open.

**Definition 1.8:** A fuzzy topological space $(X, \tau)$ is said to be fuzzy semi super normal if for every pair of fuzzy semi super closed sets $A$ and $B$ of $X$ such that $A \leq (1-B)$, there exists fuzzy semi super open sets $U$ and $V$ such that $A \leq U$, $B \leq V$ and $\text{cl}(U \cap V)$.  

**Remark 1.18:** If $F$ is fuzzy regular super closed and $A$ is fuzzy $w$-super closed sub set of a fuzzy semi super normal space $(X, \tau)$ and $(A \cap F)$ Then there exists fuzzy super open set $U$ and $V$ such that $\text{cl}(A) \leq U$, $F \leq V$ and $\text{cl}(U \cap V)$.

**Remark 1.19:** Let $A$ be a fuzzy $w$-super closed set in a fuzzy topological space $(X, \tau)$ and $f: (X, \tau) \rightarrow (Y, \sigma)$ is a fuzzy irresolute and fuzzy super closed mapping. Then $f(A)$ is fuzzy $w$-super closed in $Y$. 

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**Definition 1.9:** A collection \( \{A_i : i \in \Lambda \} \) of fuzzy w-super open sets in a fuzzy topological space \((X, \tau)\) is called a fuzzy w-super open cover of a fuzzy set \(B\) of \(X\) if \(B \subseteq \bigcup \{A_i : i \in \Lambda \}\).

**Definition 1.10:** A fuzzy topological space \((X, \tau)\) is called w-compact if every fuzzy w-super open cover of \(X\) has a finite sub cover.

**Definition 1.11:** A fuzzy set \(B\) of a fuzzy topological space \((X, \tau)\) is said to be fuzzy w-super compact relative to \(X\) if for every collection \( \{A_i : i \in \Lambda \} \) of fuzzy w-super open sub set of \(X\) such that \(B \subseteq \bigcup \{A_i : i \in \Lambda \}\) there exists a finite subset \(\Lambda_0\) of \(\Lambda\) such that \(B \subseteq \bigcup \{A_i : i \in \Lambda_0\}\).

**Definition 1.12:** A crisp subset \(B\) of a fuzzy topological space \((X, \tau)\) is said to be fuzzy w-super compact if \(B\) is fuzzy w-super compact as a fuzzy subspace of \(X\).

**Remark 1.20:** A fuzzy w-super closed crisp subset of a fuzzy w-super compact space is fuzzy w-super compact relative to \(X\).

### II. Fuzzy W-Super Continuous Mappings

The present section investigates and a new class of fuzzy mappings which contains the class of all fuzzy super continuous mappings and contained in the class of all fuzzy g-super continuous mappings.

**Definition 2.1:** A mapping \(f: (X, \tau) \to (Y, \sigma)\) is said to be fuzzy w-super continuous if the inverse image of every fuzzy super closed set of \(Y\) is fuzzy w-super closed in \(X\).

**Theorem 2.1:** A mapping \(f: (X, \tau) \to (Y, \sigma)\) is fuzzy w-super continuous if and only if the inverse image of every fuzzy super open set of \(Y\) is fuzzy w-super open in \(X\).

**Proof:** It is obvious because \(f^{-1}(1-U) = 1-f^{-1}(U)\) for every fuzzy set \(U\) of \(Y\).

**Remark 2.1:** Every fuzzy super continuous mapping is fuzzy w-super continuous, but the converse may not be true for.

**Example 2.1:** Let \(X = \{a, b\}\) and \(Y = \{x, y\}\) and the fuzzy sets \(U\) and \(V\) are defined as follows
\[
U(a) = 0.5, \quad U(b) = 0.4, \quad V(x) = 0.5, \quad V(y) = 0.5,
\]
Let \(\tau = \{0, U, 1\}\) and \(\sigma = \{0, V, 1\}\), be fuzzy topologies on \(X\) and \(Y\) respectively. Then the mapping \(f: (X, \tau) \to (Y, \sigma)\) defined by \(f(a) = x\) and \(f(b) = y\) is fuzzy w-super continuous but not fuzzy super continuous.

**Remark 2.2:** Every fuzzy w-super continuous mapping is fuzzy g-super continuous, but the converse may not be true for.

**Example 2.2:** Let \(X = \{a, b\}\) and \(Y = \{x, y\}\) and the fuzzy sets \(U\) and \(V\) are defined as follows
\[
U(a) = 0.7, \quad U(b) = 0.6, \quad V(x) = 0.6, \quad V(y) = 0.7,
\]
Let \(\tau = \{0, U, 1\}\) and \(\sigma = \{0, V, 1\}\), be fuzzy topologies on \(X\) and \(Y\) respectively. Then the mapping \(f: (X, \tau) \to (Y, \sigma)\) defined by \(f(a) = x\) and \(f(b) = y\) is fuzzy g-super continuous but not fuzzy w-super continuous.

**Remark 2.3:** Every fuzzy w-super continuous mapping is fuzzy sg-super continuous, but the converse may not be true for.

**Example 2.3:** Let \(X = \{a, b\}\) and \(Y = \{x, y\}\) and the fuzzy sets \(U\) and \(V\) are defined as follows
\[
U(a) = 0.5, \quad U(b) = 0.4, \quad V(x) = 0.5, \quad V(y) = 0.3,
\]
Let \(\tau = \{0, U, 1\}\) and \(\sigma = \{0, V, 1\}\), be fuzzy topologies on \(X\) and \(Y\) respectively. Then the mapping \(f: (X, \tau) \to (Y, \sigma)\) defined by \(f(a) = x\) and \(f(b) = y\) is fuzzy sg-super continuous but not fuzzy w-super continuous.

**Remark 2.4:** Remarks 1.4, 1.5, 1.6, 3.1, 3.2 and 3.3 reveals the following diagram of implications:
fuzzy super continuous $\Rightarrow$ fuzzy w-super continuous $\Rightarrow$ fuzzy g-super continuous

Theorem 2.2: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is fuzzy w-super continuous then for each fuzzy point $x_\beta$ of X and each fuzzy super open set $V$ of Y such that $f(x_\beta) \in V$ then there exists a fuzzy w-super open set $U$ of X such that $x_\beta \in U$ and $f(U) \subseteq V$.

Proof: Let $x_\beta$ be a fuzzy point of X and V is fuzzy super open set such that $f(x_\beta) \in V$. Put $U = f^{-1}(V)$. Then by hypothesis U is fuzzy w-super open set of X such that $x_\beta \in U$ and $f(U) = f(f^{-1}(V)) \subseteq V$.

Theorem 2.3: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is fuzzy w-super continuous then for each point $x_\beta \in X$ and each fuzzy super open set $V$ of Y such that $f(x_\beta) \in V$ then there exists a fuzzy w-super open set $U$ of X such that $x_\beta \in U$ and $f(U) \subseteq V$.

Proof: Let $x_\beta$ be a fuzzy point of X and V is fuzzy super open set such that $f(x_\beta) \in V$. Put $U = f^{-1}(V)$. Then by hypothesis U is fuzzy w-super open set of X such that $x_\beta \in U$ and $f(U) = f(f^{-1}(V)) \subseteq V$.

Definition 2.5: Let $(X, \tau)$ be a fuzzy topological space. The w-super closure of a fuzzy set $A$ of X denoted by $w\text{cl}(A)$ is defined as follows:

$$w\text{cl}(A) = \bigwedge \{ B: B \supseteq A, B \text{ is fuzzy w-super closed set in } X \}$$

Remark 2.3: It is clear that, $A \subseteq g\text{cl}(A) \subseteq w\text{cl}(A) \subseteq \text{cl}(A)$ for any fuzzy set $A$ of $X$.

Theorem 2.4: A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is fuzzy w-super continuous then $f(w\text{cl}(A)) \subseteq \text{cl}(f(A))$ for every fuzzy set $A$ of $X$.

Proof: Let A be a fuzzy set of X. Then $\text{cl}(f(A))$ is a fuzzy super closed set of Y. Since $f$ is fuzzy w-super continuous $f^{-1}(\text{cl}(f(A)))$ is fuzzy w-super closed in X. Clearly $A \subseteq f^{-1}(\text{cl}(f(A)))$. Therefore $w\text{cl}(A) \subseteq w\text{cl}(f^{-1}(\text{cl}(f(A)))) = f^{-1}(\text{cl}(f(A)))$. Hence $f(w\text{cl}(A)) \subseteq \text{cl}(f(A))$.

Definition 2.6: A fuzzy topological space $(X, \tau)$ is said to be fuzzy $w\text{-}T_{1/2}$ if every fuzzy w-super closed set in X is fuzzy semi super closed.

Theorem 2.5: A mapping $f$ from a fuzzy $w\text{-}T_{1/2}$ space $(X, \tau)$ to a fuzzy topological space $(Y, \sigma)$ is fuzzy super continuous if and only if it is fuzzy w-super continuous.

Proof: Obvious.

Remark 2.4: The composition of two fuzzy w-super continuous mappings may not be fuzzy w-super continuous. For

Example 2.2: Let $X = \{a, b\}$, $Y = \{x, y\}$, $Z = \{p, q\}$ and the fuzzy sets $U, V$ and $W$ defined as follows: $U(a) = 0.5$, $U(b) = 0.4$, $V(x) = 0.5$, $V(y) = 0.3$, $W(p) = 0.6$, $W(q) = 0.4$. Let $\tau = \{0, U, 1\}$ and $\sigma = \{0, V, 1\}$ and $\mu = \{0, W, 1\}$ be the fuzzy topologies on $X, Y$ and $Z$ respectively. Then the mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ defined by $f(a) = x$ and $f(b) = y$ and the mapping $g: (Y, \sigma) \rightarrow (Z, \mu)$ defined by $g(x) = p$, and $g(y) = q$. Then $f$ and $g$ are w-super continuous but $gof$ is not fuzzy w-super continuous. However,

Theorem 2.6: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is fuzzy w-super continuous and $g: (Y, \sigma) \rightarrow (Z, \mu)$ is fuzzy super continuous. Then $gof: (X, \tau) \rightarrow (Z, \mu)$ is fuzzy w-super continuous.
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**Proof:** Let A be a fuzzy super closed in Z then \( g^{-1}(A) \) is fuzzy super closed in Y, because g is super continuous .Therefore \((gof)^{-1}(A) = f^{-1}(g^{-1}(A))\) is fuzzy w-super closed in X. Hence gof is fuzzy w-super continuous.

**Theorem 2.7:** If \( f:(X,\tau)\rightarrow(Y,\sigma) \) is fuzzy w-super continuous and \( g:(Y,\sigma)\rightarrow(Z,\mu) \) is fuzzy super continuous is fuzzy g-super continuous and \( (Y,\sigma) \) is fuzzy T\(_{1/2}\) then gof: \( (X,\tau)\rightarrow(Z,\mu) \) is fuzzy w-super continuous.

**Proof:** Let A be a fuzzy super closed set in Z then \( g^{-1}(A) \) is fuzzy g-super closed set in Y because g is g-super continuous. Since Y is T\(_{1/2}\), \( g^{-1}(A) \) is fuzzy super closed in Y. And so, \((gof)^{-1}(A) = f^{-1}(g^{-1}(A))\) is fuzzy w-super closed in X. Hence gof: \( (X,\tau)\rightarrow(Z,\mu) \) is fuzzy w-super continuous.

**Theorem 2.8:** A fuzzy w-super continuous image of a fuzzy w-compact space is fuzzy compact.

**Proof:** Let f: \( (X,\tau)\rightarrow(Y,\sigma) \) is a fuzzy w-super continuous mapping from a fuzzy w-compact space \( (X,\tau) \) on to a fuzzy topological space \( (Y,\sigma) \). Let \{\( A_i \): i \in \( A \)\} be a fuzzy super open cover of Y. Then \{\( f^{-1}(A_i) \): i \in \( A \)\} is a fuzzy w-super open cover of X. Since X is fuzzy w-compact it has a finite fuzzy sub cover say \( f^{-1}(A_1), f^{-1}(A_2), \ldots, f^{-1}(A_n) \). Since f is on to \{\( A_1,A_2,\ldots,A_n \)\} is an open super cover of Y. Hence \( (Y,\sigma) \) is fuzzy compact.

**Definition 2.7:** A fuzzy topological space X is said to be fuzzy w-super compact if there is no proper fuzzy set of X which is both fuzzy w-super open and fuzzy w-super closed.

**Remark 2.5:** Every fuzzy w-super connected space is fuzzy super connected, but the converse may not be true for the fuzzy topological space \( (X,\tau) \) in example 2.1 is fuzzy super connected but not fuzzy w-super connected.

**Theorem 2.9:** If \( f:(X,\tau)\rightarrow(Y,\sigma) \) is a fuzzy w-super continuous surjection and X is fuzzy w-super connected then Y is fuzzy super connected.

**Proof:** Suppose Y is not fuzzy super connected. Then there exists a proper fuzzy set G of Y which is both fuzzy super open and fuzzy super closed. Therefore \( f^{-1}(G) \) is a proper fuzzy set of X, which is both fuzzy w-super open and fuzzy w-super closed in X, because f is fuzzy w-super continuous surjection. Hence X is not fuzzy w-connected, which is a contradiction.

**Reference**


