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Investigation of lateral-directional static stability of the aircraft Y

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Abstract: *The study of lateral static stability of aircraft in general and aircraft Y in particular is relatively complex; solving this problem requires the use of linear equations systems combined with modern computational methods. Solving the lateral-directional static stability problem for aircraft Y will contribute to maximizing the tactical and technical capabilities of this type of aircraft, providing a basis for successfully accomplishing combat missions using air firepower.*
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I. INTRODUCTION

Aircraft Y is a multirole fighter aircraft featuring good operational and tactical performance and a long range. It is currently operated by air forces in many countries around the world. Operational experience has shown that the operational and tactical capabilities of this aircraft have not yet been fully exploited. One of the key requirements for a fighter aircraft is stability and control, which determine its maneuverability, survivability, and mission accomplishment capability.

To solve the stability problem, it is necessary to formulate a multi-parameter system of equations and linearize it. The resulting system of differential equations is then solved using numerical methods. In this study, the Runge–Kutta method is employed.

II. METHODOLOGY AND PROBLEM FORMULATION

2.1. Initial assumptions

In this problem, the following assumptions are adopted: The aircraft is performing steady, level flight at the time before the control stick is deflected, the aircraft is a perfectly rigid body, all forces acting on the aircraft are applied at the center of gravity, and the sum of all moments acting on the aircraft is equal to zero, the ground within the region considered for the aircraft motion is flat, gravitational acceleration there is assumed constant, and the moments of inertia I_x, I_y, I_z about the axes of the body-fixed coordinate system are the principal moments of inertia of the aircraft.

The process of aileron deflection is assumed to be instantaneous, and the time required to attain the bank angle is short; the transient process during control is therefore neglected. The analysis time is short, so the altitude may be considered constant and the change in aircraft weight negligible. Thus, in this problem, the following are known:

- Aircraft parameters such as: $m, L_{mb}, l_c, b_a, \chi, S, I_x, I_y, I_z$;

- Parameters of the initial flight condition of the aircraft such as: altitude *H*, engine operating condition ($n = const$), initial sideslip angle β_0, initial bank angle γ_0, aileron and rudder deflection angles $\delta_l, \delta_H = 0$;

- Atmospheric environment: density and speed-of-sound functions depending on altitude (according to the standard atmosphere table);

- The aircraft has the following prescribed aerodynamic characteristics: $C_{x0} = C_{x0}(M)$, $A = A(M)$, $m_x^{\omega y}(M)$, $m_y^{\omega y}(M)$. The engine has speed and altitude characteristics $P = P(V, H)$.

2.2. General equations of motion

The motion of the aircraft in space as a perfectly rigid body is determined by a system of 12 equations as follows [1]:

- Equations of motion of the center of mass:

$$m\frac{dV}{dt} = P\,cos(\alpha + \alpha_P) - X - G\,sin\,\theta\,; \tag{1}$$

$$mV\frac{d\theta}{dt} = P\,sin(\alpha + \alpha_P)cos\,\gamma + Y\,cos\,\gamma - G\,cos\,\theta\,; \tag{2}$$

$$-mV\,cos\,\theta\frac{d\varphi}{dt} = P\,sin(\alpha + \alpha_P)sin\,\gamma + Y\,sin\,\gamma\,; \tag{3}$$

where: m – aircraft mass, V – velocity, P – engine thrust, α – angle of attack, α_P – engine installation angle, θ – flight-path inclination angle, γ – aircraft bank angle, φ – flight-path rotation angle, X – drag, Y – lift, G – gravity force.

- Equations of rotational motion of a rigid body about the center of mass:

$$I_x\frac{d\omega_x}{dt} + (I_z - I_y)\omega_z\omega_y = m_x\frac{\rho V^2}{2}Sl\,; \tag{4}$$

$$I_y\frac{d\omega_y}{dt} + (I_x - I_z)\omega_x\omega_z = m_y\frac{\rho V^2}{2}Sl\,; \tag{5}$$

$$I_z\frac{d\omega_z}{dt} + (I_y - I_x)\omega_y\omega_x = m_z\frac{\rho V^2}{2}Sb_a\,; \tag{4}$$

where: I_x, I_y, I_z - moments of inertia of the aircraft corresponding to the axes O_{x1}, O_{y1}, O_{z1}; m_x, m_y, m_z – moment coefficients; ρ – air density; S – wing area; l – wingspan; b_a – mean aerodynamic chord; ω_y - the projection of the angular velocity of flight-path rotation onto an axis of the velocity coordinate system, $\omega_y = \frac{d\varphi}{dt}cos\theta$ while ω_x, ω_z are the projections of the angular velocity of aircraft rotation onto the axes of the body-fixed coordinate system.

- Euler kinematic equations:

$$\frac{d\vartheta}{dt} = \omega_y\,sin\,\gamma + \omega_z\,cos\,\gamma\,; \tag{5}$$

$$\frac{d\gamma}{dt} = \omega_x - (\omega_y\,cos\,\gamma - \omega_z\,sin\,\gamma)tg\,\vartheta\,; \tag{6}$$

$$\frac{d\psi}{dt} = \frac{(\omega_y\,cos\,\gamma - \omega_z\,sin\,\gamma)}{cos\,\vartheta}\,; \tag{7}$$

where: ϑ pitch angle; ψ flight heading angle.

2.3. Lateral-motion equations

In the velocity coordinate system, the components $V_y = 0$, $V_z = 0$ while $V_x = V$. To determine the system of lateral-motion equations, impose the condition $\omega_z = 0$ and omit the longitudinal-motion equations from the general system of equations; we obtain the following system:

$$-m\omega_y V = \sum F_z \,; \tag{8}$$

$$I_x \frac{d\omega_x}{dt} = \sum M_x \,;\; I_y \frac{d\omega_y}{dt} = \sum M_y \,; \tag{9}$$

$$\frac{d\gamma}{dt} = \omega_x - \omega_y \cos\gamma tg\vartheta \,;\; \frac{d\psi}{dt} = \omega_y \frac{\cos\gamma}{\cos\vartheta}; \tag{10}$$

where the first equation is written in the vertical velocity coordinate system; it characterizes the translational motion of the center of mass in the horizontal plane. The next two equations are written in the body-fixed coordinate system and characterize the rotational motion of the aircraft. The last two equations are the kinematic equations, F_z - resultant force in longitudinal motion.

2.4. Linearization of the lateral-motion equations

The system of lateral-motion equations of the aircraft, after linearization, becomes the following system of equations [1]:

$$mV\left(\frac{d\beta}{dt} - \frac{d\psi}{dt}\right) + Z^{\beta}\beta - Y\gamma = Z^{\delta_H}\delta_H \,; \tag{11}$$

$$I_x \frac{d\omega_x}{dt} - M_x^{\beta}\beta - M_x^{\omega_x}\omega_x - M_x^{\omega_y}\omega_y = M_x^{\delta_H}\delta_H + M_x^{\delta_l}\delta_l \,; \tag{12}$$

$$I_y \frac{d\omega_y}{dt} - M_y^{\beta}\beta - M_y^{\omega_x}\omega_x - M_y^{\omega_y}\omega_y = M_y^{\delta_H}\delta_H + M_y^{\delta_l}\delta_l \,; \tag{13}$$

$$\frac{d\gamma}{dt} = \omega_x \,;\; \frac{d\psi}{dt} = \omega_y \,. \tag{14}$$

This is a system of nonhomogeneous linear differential equations containing functions that describe changes in the parameters of longitudinal motion: $\beta(t)$, $\psi(t)$, $\gamma(t)$, $\omega_x(t)$, $\omega_y(t)$. The right-hand side of the system contains functions that describe changes in the rudder deflection angle $\delta_H(t)$ and the aileron deflection angle $\delta_l(t)$.

2.5. Determination of sideslip angle at different Mach numbers and altitudes

Because the aircraft is in level flight, we may take $\frac{d\varphi}{dt} = 0$, because the rotational motion of the aircraft occurs much faster than the curvature of the flight path, $\frac{d\beta}{dt} - \frac{d\psi}{dt} = 0$, substituting this into equation 11 gives an ordinary algebraic equation, which may be omitted. We may also neglect the torsional moments $-\frac{M_x^{\omega_y}}{I_x}$ and $-\frac{M_x^{\omega_x}}{I_x}$, in which case equations 12, 13 take the following form [1]:

$$\frac{d^2\gamma}{dt} + n_{2\omega_X}\frac{d\gamma}{dt} + n_{2\beta}\beta = 0\,; \tag{15}$$

$$\frac{d^2\beta}{dt} + n_{2\omega_y}\frac{d\beta}{dt} + n_{3\beta}\beta = 0\,; \tag{16}$$

Solving the system of equations (15, 16), we obtain the formula for determining the sideslip angle:

$$\beta = \frac{\beta_0}{\sin\varphi_\beta} e^{-n_{oc}t} \sin\left(\omega_c t + \varphi_\beta\right). \tag{17}$$

And the formula for determining the bank angle:

$$\gamma = C_1 e^{-n_{oc}t} \sin\left(\omega_c t + \varphi_\gamma\right) + C_3 e^{-n_{\omega 2x}t} + C_4\,. \tag{18}$$

where, $n_{0c} = -\frac{1}{2}\frac{M_y^{\omega_y}}{I_y}$, $n_{0c} = -\frac{M_x^{\omega_x}}{I_y}$, the oscillation frequency$\omega_c$is determined as follows:

$$\omega_c = \sqrt{\frac{M_y^{\beta}}{I_y} - \frac{1}{4}\left(\frac{M_y^{\omega_y}}{I_y}\right)^2}\,. \tag{19}$$

The constants C_1, C_3, C_4, φ_γ are determined from the initial conditions.

III. LATERAL-DIRECTIONAL STABILITY CALCULATION

3.1. Determination of problem parameters

Assume that in the initial condition, the aircraft Y [2] carries no externally mounted weapons, is in steady level flight, and has the following parameters:

- Mass: $m = 24000$ *(kg)*,
- The engine operates at rotational speed: $n = 85\%$,
- Altitude: $H = 5000$ *(m)*,
- Gravitational acceleration: $g = 9{,}81$ *(m/s²)*,
- Speed of sound: $a = 321$ *(m/s)*,
- Atmospheric pressure: $p = 54045$ (N/m^2),
- Air density: $\rho = 0{,}73654$ (kg/m^3),
- Wing area $S = 62$ (m^2),
- Wingspan: $l_c = 14{,}7$ *(m)*,
- Aircraft length: $l_{mb} = 21{,}9$ *(m)*,
- Sweep angle: $\chi = 42^0$.

Using the Rungle-Kutta method and the Matlab programming language to build the calculation program, we obtain the following results:

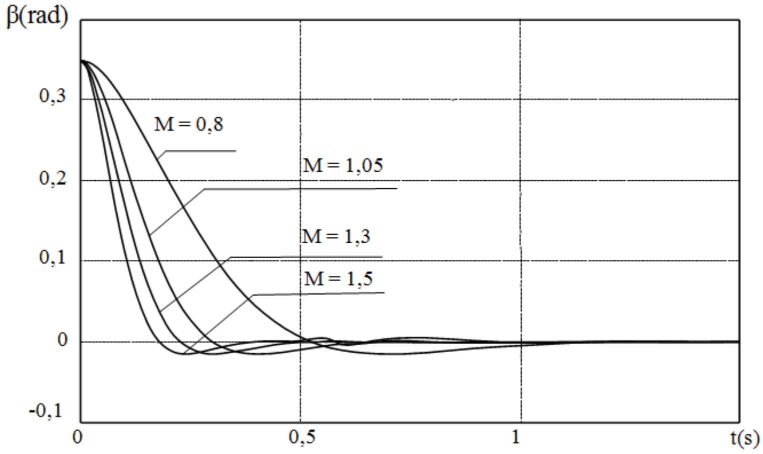

Figure 1. Variation of the sideslip angle during motion as the Mach number (M) changes

Thus, suppose the aircraft is in steady level flight with sideslip angle $\beta = 0$, then a disturbance occurs and produces sideslip; at the initial instant when the sideslip appears, the aircraft turns to the left by an angle β_0, and the angular rotation rate $\omega = 0$. Due to the effect of the moment $(M_y)_\beta < 0$ the sideslip angle β undergoes damped oscillation, and after a period of time the sideslip angle becomes 0.

At different Mach numbers, the time and amplitude of the damped oscillation are also different. As the Mach number increases, the oscillation time of the angle β decreases, which means that at higher Mach numbers the aircraft has greater stability.

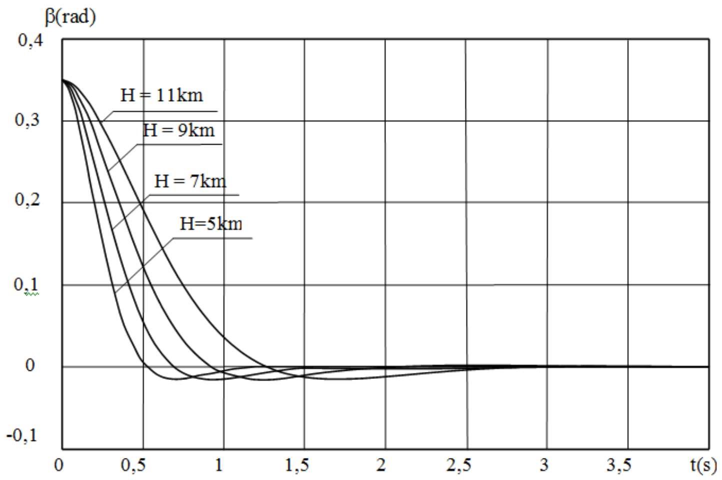

Figure 2. Variation of the sideslip angle during aircraft motion at different altitudes under disturbance

Thus, the aircraft is in steady level flight; if a disturbance occurs with an initial sideslip angle β_0, the higher the altitude, the longer the damped-oscillation time, which shows that when flying at higher altitude and encountering a disturbance, the aircraft will be less stable.

For the bank angle, the results are as follows:

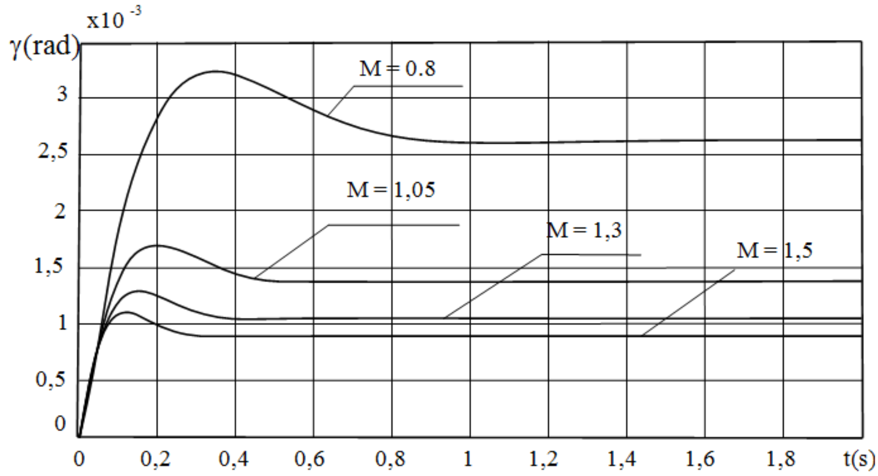

Figure 3. Variation of the bank angle during motion at different Mach numbers

From the graph above, it can be seen that when the aircraft experiences sideslip with angle β_0, the aircraft has angular velocity $\omega_y < 0$ producing a lateral torsional moment that causes the aircraft to bank to the right. From that moment, rolling motion begins with an increasing bank angle. Thus, stability with respect to the bank angle does not ensure flight-direction stability; the motion after the disturbance has a flight direction different from the initial direction.

At higher Mach numbers, the decay time is shorter and the value of the bank angle when it becomes constant is smaller.

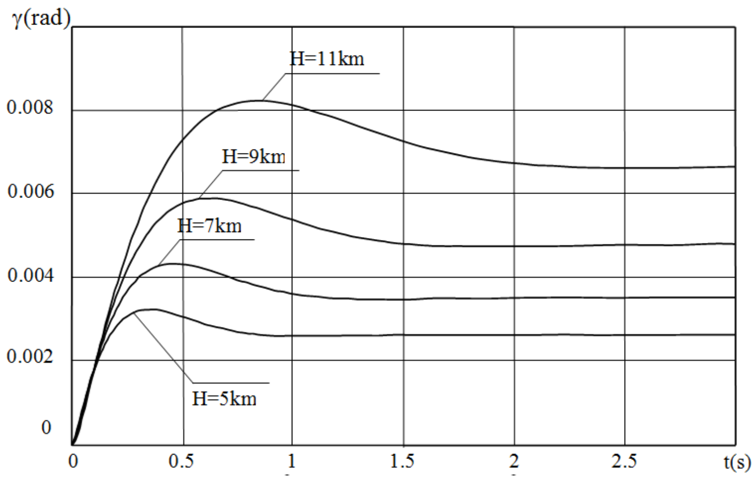

Figure 4. Variation of the bank angle during aircraft motion at different altitudes under disturbance

Thus, when the aircraft encounters a lateral disturbance during motion, at different altitudes the bank angle does not return stably to the initial position and the damped-oscillation times are different. The lower the altitude, the shorter the damping time of the bank-angle oscillation.

IV. CONCLUSION

The lateral stability problem of the Y aircraft is relatively complex; solving several separate lateral-stability problems as above yields calculation results consistent with natural laws. The research results can be used to evaluate the lateral stability of the aircraft in certain specific cases, and can be used to assess its tactical and technical characteristics, providing a basis for effective operation of this type of aircraft.

Conflict of interest
There is no conflict to disclose.

REFERENCES

[1] Cook, M. V. Flight Dynamics Principles: A Linear Systems Approach to Aircraft Stability and Control. 3rd ed. Butterworth-Heinemann/Elsevier, 2012.
[2] Etkin, B., Reid, L. D. Dynamics of Flight: Stability and Control. 3rd ed. John Wiley & Sons, 1996.
[3] Лазнюк П.С., Максимов В.С. и др. Динамика полета. Методические указания по выполнению курсовой работы. К.КИИГА, 1987.
[4] Мхитарян А.М., Лазнюк П.С. и др. Динамика полета: Учебник для авиационных вузов. 2-е изд. Перераб. и доп. М.: Машиностроение, 1998.
[5] Мелкумов Пугачев Н.С. Теория авиационных поршневых двигателей. М.: ВВИА им. Жуковского, 1993.