

Calculation of the target destruction area when bombing at low altitude based on flight trajectory

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Abstract: This paper discusses the determination of the destruction zone of a bomb when launched at low altitude. For the convenience of research, the safety zone is divided into an absolute safety zone and a relative safety zone. The ballistic curve model begins with the analysis of the projectile's trajectory, followed by an examination of the forces acting on the projectile in flight. Using Newton's second law and differential equations, the ballistic curve model is established to describe the projectile's trajectory. After that, the trajectory curve parameters of the given launch angle are calculated by using MATLAB software, and the farthest range of the projectile is obtained. According to the relationship between the range and the angle of the projectile in the ballistic curve model, the angle of the projectile can be determined. Then, by combining this angle with the upper limit of the projectile launching error, the range of the projectile's falling point can be calculated.

Keywords: destruction zone of bomb, launching at low altitude, ballistic curve.

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I. INTRODUCTION

The problem of establishing a designated area for destruction must be considered when a bomb is launched. Taking the projectile's launching speed as an example $v_0 = 200 \text{ m/s}$, the launching angle is not limited; it is necessary to determine the destruction area of the launching exercise by analyzing the trajectory curve.

For the determination of the destruction zone, it is obvious that we should start from the killing range of the shell. The killing range of a shell is determined by the range, killing radius, firing error, and other parameters. Among them, the killing radius and launching error of the shell are determined by the performance of the weapon itself, so the determination of the range of the shell is the key to solving this problem. It is clear that beyond the maximum range is the absolute safety zone. But in the actual launching performance, it will not use the farthest range of the shell to hit the target, because the launching accuracy will be very poor. Therefore, the relative safety zone should also be considered when the weapon is aimed at a specific target within its range.

Based on Newton's second law, the differential equation describing the ballistic curve of a projectile can be established. Then, the functional relationship between the range and the angle of the projectile is derived by using the equation. Thus, the launching angle corresponding to the farthest range and any range can be obtained. Combined with the upper limit of launching error angle, the absolute safety zone and relative safety zone of the bomb launching exercise can be determined.

II. THE ESTABLISHMENT OF THE MODEL

2.1. The Establishment of Ballistic Curve Model

It is assumed that the flying projectile is an ideal particle, and the air resistance is only related to the velocity and not affected by the wind. Analyze the force of its operation in the air, as shown in the following figure.

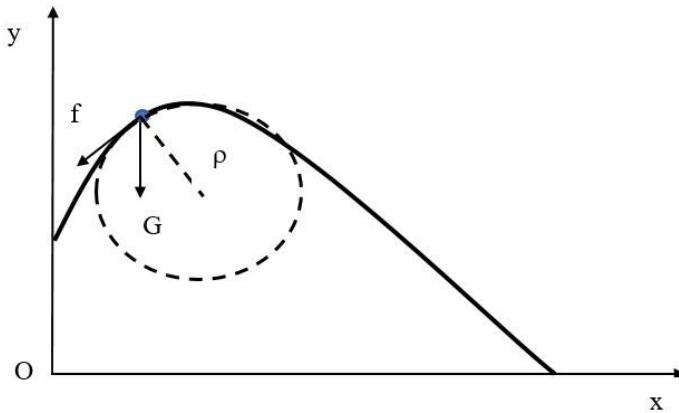


Figure 1. The analysis of the shell force

In Figure 1, f is the resistance, θ is the angle between the shell speed and the horizontal direction, and ρ is the radius of curvature of the shell trajectory. By analyzing the above figure, considering the acceleration in tangent direction and centripetal acceleration in normal direction of the projectile trajectory, the ballistic differential equation model can be obtained:

$$\begin{cases} m \frac{dV}{dt} = -f(V) - mg \sin \theta \\ m \frac{V^2}{\rho} = mg \cos \theta \end{cases} \quad (1)$$

Considering the calculation method of air resistance, as referenced in [1, 3], when the velocity of the projectile is less than a certain value, it is generally considered that air resistance is directly proportional to the square of the velocity.

Therefore:

$$f(V) = kV^2 \quad (2)$$

It can be solved by combining (1) and (2):

$$V^2 = \cos^2 \theta \left[C - \frac{1}{2} K \ln \frac{1 + \sin \theta}{1 - \sin \theta} \right] - K \sin \theta \quad (3)$$

$$\frac{dx}{d\theta} = \frac{V^2}{g}, \frac{dy}{d\theta} = \frac{V^2 \tan \theta}{y}, \frac{dt}{d\theta} = \frac{V \sin \theta}{g} \quad (4)$$

$$x = \int_{\theta_1}^{\theta_0} \frac{1}{g} \left[\cos^2 \theta \left(C - \frac{1}{2} K \ln \left(\frac{1 + \sin \theta}{1 - \sin \theta} \right) - K \sin \theta \right) \right] d\theta \quad (5)$$

$$y = \int_{\theta_1}^{\theta_0} \frac{\tan \theta}{g} \left[\cos^2 \theta \left(C - \frac{1}{2} K \ln \left(\frac{1 + \sin \theta}{1 - \sin \theta} \right) - K \sin \theta \right) \right] d\theta \quad (6)$$

$$t = \int_{\theta_1}^{\theta_0} \frac{\sin \theta}{g} \sqrt{\cos^2 \theta \left(C - \frac{1}{2} K \ln \left(\frac{1 + \sin \theta}{1 - \sin \theta} \right) - K \sin \theta \right)} d\theta \quad (7)$$

With: θ_1 is the angle between the instantaneous velocity direction of the shell and the horizontal line, and θ_0 is the launching angle.

2.2. The Establishment of Destruction Zone Model

Considering the problem of the safety area in launching exercise, a plane rectangular coordinate system is established, which takes the position of the airplane as the origin and the line between the bomb and target as the x-axis. The area beyond the maximum damage range of the shell is safe. During the actual launching exercise, the shell aims at a certain target within the range. The bomb determines the launching angle according to the target

point, and the launching angle determined by the bomb must have an error, which will cause the shell to fall in an area around the target point. This area, together with the shell damage extension area, is an unsafe area. As shown in the figure below:

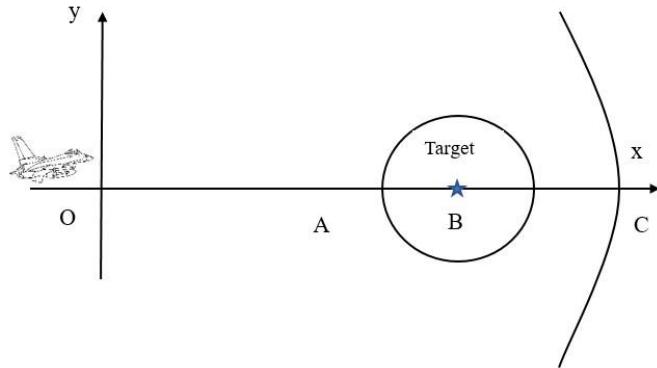


Figure 2. The schematic diagram of each area

Where:
 A - Relative safety zone;
 B - Hazardous area;
 C - Absolutely a safety zone.

The hazardous area is inside the circle with the origin as the center and $L + r$ as the radius. is the maximum range, which can be obtained from the ballistic curve model, and is the killing radius of the shell. In the coordinate system, the hazardous area can be expressed as:

$$x^2 + y^2 \leq (L + r)^2 \quad (8)$$

The absolute safe area can be expressed as:

$$x^2 + y^2 > (L + r)^2 \quad (9)$$

For the target point at x_0 meter from the airplane aiming line, substitute the data into the ballistic curve models (1) and (2), and the theoretically corresponding launching angle θ can be obtained. Considering the aiming error δ , the maximum offset target distance can be obtained by using the ballistic curve model:

$$l = \max \{X(\theta + \delta) - x_0, X(\theta - \delta) - x_0\} \quad (10)$$

The distance when launching angle is θ , which can be calculated by formula (4).

According to formula (10), the circle with x_0 as the center and $l + r$ as the radius is the dangerous area:

$$(x - x_0)^2 + y^2 < (l + r)^2 \quad (11)$$

Removing this area and the absolute safety area is the relative safety area. Its expression is:

$$\begin{cases} (x - x_0)^2 + y^2 > (l + r)^2 \\ x^2 + y^2 < (L + r)^2 \end{cases} \quad (12)$$

In there, formulas (10) and (12) are the safe area models.

III. THE SOLUTION OF SOFTWARE SIMULATION

It is difficult to solve the differential equation in the trajectory curve model, and it can not get the explicit solution of the trajectory equation. In this paper, MATLAB software is used to program the numerical solution of the differential equation (refer to the data in [2], take the air resistance coefficient).

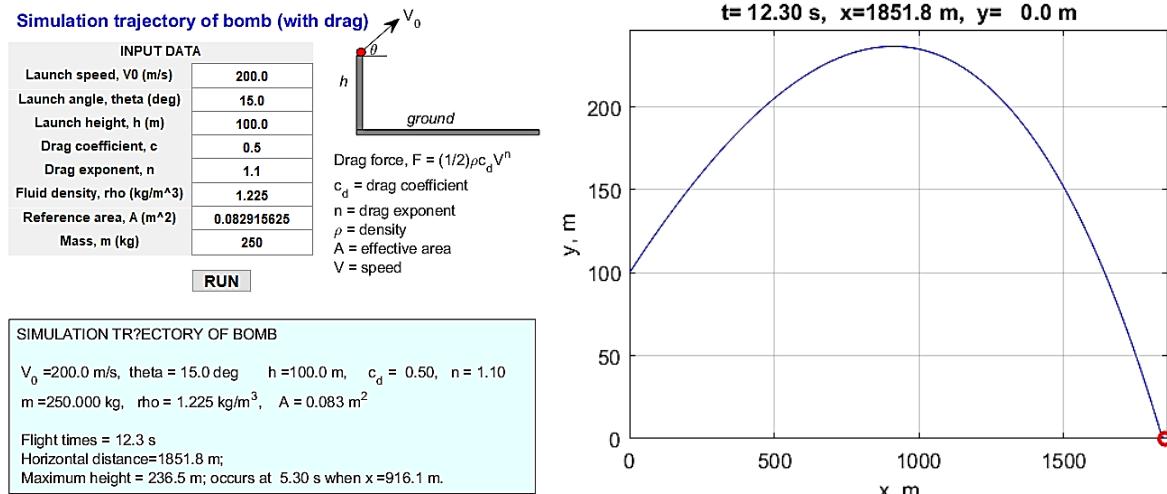


Figure 3. Simulation trajectory of a bomb

Figure 3 simulates the trajectory of the ОФАБ-250 bomb when launching at low altitude (100m) with an aircraft speed of 200m / s .

From the program to calculate the bomb's flight trajectory described in Figure 3. By changing the bombing angle, we get the area where the bomb can destroy the target when bombing at low altitude, as shown in Figure 4.

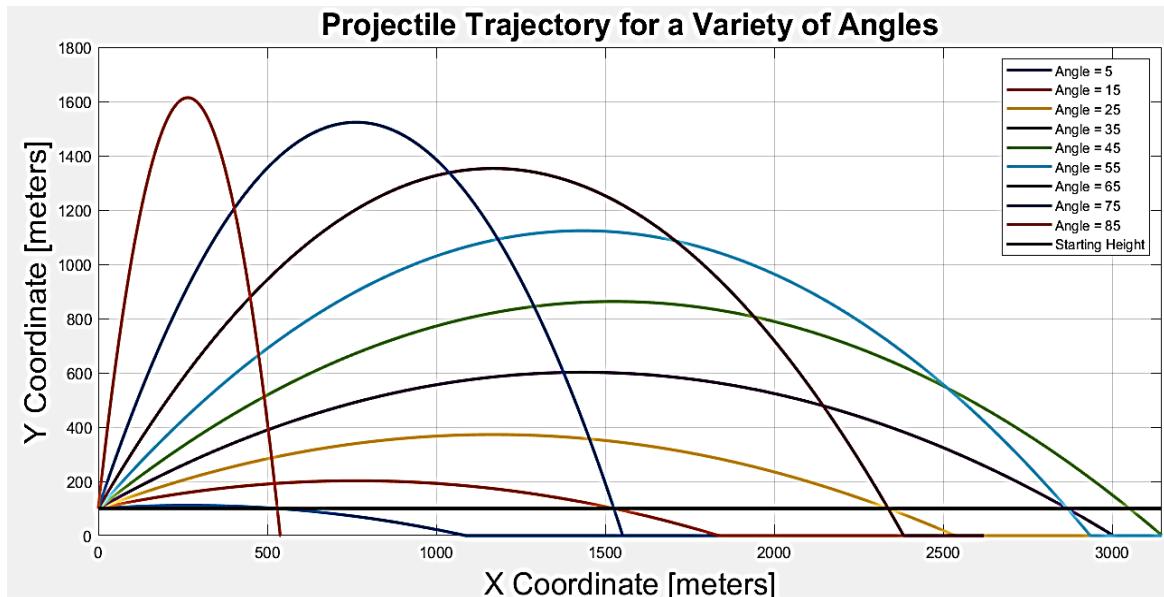


Figure 4. Hazardous area of a bomb with another launching angle

IV. EVALUATION OF THE MODEL

It is an important tactical calculation in bomb launching to determine the hazardous zone. In this paper, the research on the hazardous area of bomb acting can be solved by computer, which can meet the requirement of automatic calculation, and solve the problem of accurately determining the hazardous area in theory, which provides an effective calculation method for the development of bomb launching on an airplane.

From the above model, we can determine the area where the target can be destroyed when bombing at low altitude. Contributing to improving combat effectiveness in reality.

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