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Synthesize an effective, simple guidance law for the "air-to-ground" flight device

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Abstract: *The paper presents a method to synthesize the optimal guidance law for an "air to ground" flight device with the requirement of a simple guidance law and ensure accurate guidance to the target when large initial deviation angle. The guidance law is built on the basis of the optimal control theory according to the optimal energy criterion, with a nonlinear flight device-target kinetic dynamic model. The simulation results demonstrate the superiority of the guiding law applied to the flight device delivered to the ground target.*
Keywords: *flight device, optimal control theory.*

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I. INTRODUCTION

The proportional navigation guidance law has many advantages such as high accuracy, which can be applied when bringing a flight device (FD) to a target with high speed and maneuverability.

However, complex homing guidance is required, thereby increasing the mass with homing guidance and reducing the useful load (e.g., explosives...).

For a stationary or moving slowly target, it is required to synthesize a guidance law to bring the FD to the target (T) that ensures the following factors:

- Make sure to bring the FD to the T correctly, even if the initial deviation angle of the FD-MT is large.
- Bring FD to T with the maximum useful load.

With the above requirement, the paper researches a method to synthesize a simple and effective guidance law to bring the FD to the target on the ground to ensure the requirements.

The paper is divided into 5 parts. Part 2 comes from the nonlinear FD-T kinetic model, builds the equation of state. The guidance law will be synthesized in part 3 based on the optimal control theory with the criterion of minimizing energy. Numerical simulation results are given in section 4, and the conclusion of the paper will be presented in section 5.

II. PROBLEM FLIGHT DEVICE-TARGET KINEMATICS

2.1. Theoretical Foundations Problem Formulation

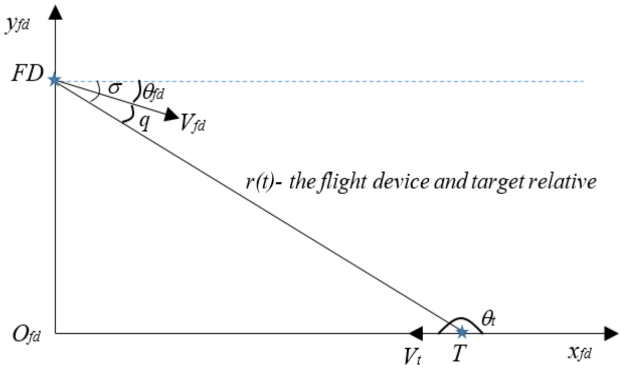

Figure 1. Planar-engagement geometry between FD-T

The control of FD in space can be done through the control of FD in two perpendicular planes [1, 2, 3]. Therefore, the navigation problem in space can be to the problem of navigation in the plane. Assuming that FD-T is considered as a point, we have a planar-engagement geometry between FD-T in the vertical plane, as shown in Figure 1.

Where, FD and T denote the flight device and target, respectively; $O_{fd}x_{fd}y_{fd}$ - The mobile ground coordinate system, the center may not coincide with the center of gravity FD; V_{fd}, ϑ_{fd} - The flight device velocity and flight path angle; q - Look angle; V_{mt}, ϑ_{mt} - The target velocity and target angle; *σ, r(t)*- Line-of-sight (LOS) angle and the flight device and target relative distance; $V_{tbb},\ V_{mt} = const$

2.2. Kinematic model building

From Figure 1, kinematic equations can be derived as follows:

$$\begin{cases} \dot{r} = -V_{fd}\cos(\vartheta_{fd}-\sigma) + V_t\cos(\vartheta_t-\sigma) \\ r\dot{\sigma} = -V_{fd}\sin(\vartheta_{fd}-\sigma) + V_t\sin(\vartheta_t-\sigma) \end{cases} \tag{1}$$

First derivatives of the second equation in (1) are given as:

$$r\ddot{\sigma} + \dot{r}\dot{\sigma} = -V_{fd}(\dot{\vartheta}_{fd}-\dot{\sigma})\cos(\vartheta_{fd}-\sigma) + V_t(\dot{\vartheta}_t-\dot{\sigma})\cos(\vartheta_t-\sigma)$$

$$r\ddot{\sigma} + \dot{r}\dot{\sigma} = -\dot{\sigma}\left(-V_{fd}\cos(\vartheta_{fd}-\sigma) + V_t\cos(\vartheta_t-\sigma)\right) - V_{fd}\dot{\vartheta}_{fd}\cos(\vartheta_{fd}-\sigma) + V_t\dot{\vartheta}_t\cos(\vartheta_t-\sigma)$$

Substitute the first equation of equations (1) into, and reevaluate:

$$r\ddot{\sigma} + \dot{r}\dot{\sigma} = -\dot{r}\dot{\sigma} - V_{fd}\dot{\vartheta}_{fd}\cos(\vartheta_{fd}-\sigma) + V_t\dot{\vartheta}_t\cos(\vartheta_t-\sigma)$$

Assuming T moves in a straight line, we have $\dot{\vartheta}_t = 0$. This assumption is completely acceptable because: Target indication information only before launch; Ground target speed is small (10÷20 *m/s*).

With the above assumption, we get:

$$\ddot{\sigma} = -2\frac{\dot{r}}{r}\dot{\sigma} - \frac{V_{fd}\dot{\vartheta}_{fd}\cos(\vartheta_{fd}-\sigma)}{r} \tag{2}$$

Let: $V = \dot{r};\ \dot{\sigma} = \zeta$. From (2) we have:

$$\begin{cases} \dot{\sigma} = \zeta \\ \dot{\zeta} = -2\dfrac{V}{r}\zeta - \dfrac{V_{fd}\dot{\vartheta}_{fd}\cos(\vartheta_{fd}-\sigma)}{r} \end{cases} \tag{3}$$

Combining (3) with $\dot{\vartheta}_{fd} = \dfrac{g}{V_{fd}}n_z$, we received:

$$\begin{cases} \dot{\vartheta}_{fd} = \dfrac{g}{V_{fd}}n_z \\ \dot{\sigma} = \zeta \\ \dot{\zeta} = -2\dfrac{V}{r}\zeta - \dfrac{g\cos(\vartheta_{fd}-\sigma)n_z}{r} \end{cases} \tag{4}$$

The boundary condition at the end is:

- FD-T distance approaches 0; Small line-of-sight angle speed ($\dot{\sigma} \to 0$).

Thus, the constraint condition at the control process endpoint: Speed component for turning the line of sight $\left(V_{fd}\sin(\vartheta_{fd}-\sigma) - V_t\sin(\vartheta_t-\sigma)\right)\Big|_{t_f} = 0$. In which, t_f is the end time of the control process, which is understood as the time when the flight device hits the T. If we choose t_f a smaller t_f hits the T, then after t_f, the flight device's distance increases, the FD-T's distance decreases to "0". So the first requirement does not need to be considered.

Assume $\vartheta_{fd}-\sigma$ and $\vartheta_t-\sigma$ take the following values: $\sin(\vartheta_{fd}-\sigma)\big|_{t_f} \approx (\vartheta_{fd}-\sigma)\big|_{t_f}$; $\sin(\vartheta_t-\sigma)\big|_{t_f} \approx (\vartheta_t-\sigma)\big|_{t_f}$.

Get the boundary condition:

$$\left(V_{fd}(\vartheta_{fd}-\sigma) - V_t(\vartheta_t-\sigma)\right)\Big|_{t_f} = 0$$

$$\Leftrightarrow \left((\vartheta_{fd}-\vartheta_0) - (1-k_V)\sigma\right)\Big|_{t_f} = 0$$

We get:

$$\left(\Delta\vartheta + (1-k_V)\sigma\right)\Big|_{t_f} = 0 \tag{5}$$

Where, $k_V = V_t / \mathrm{V}_{fd}$; $\Delta\vartheta = \vartheta_0 - \vartheta_{fd}$; $\vartheta_0 = k_V\vartheta_t$

Received: $\Delta\dot{\vartheta} = -\dot{\vartheta}_{fd}$

We get:

$$\varphi = \Delta\vartheta + (1-k_V)\sigma \Rightarrow \dot{\varphi} = \Delta\dot{\vartheta} + (1-k_V)\dot{\sigma} \tag{6}$$

We can rewrite in the form (4) as follows:

$$\begin{cases} \dot{\varphi} = \left(1-k_V\right)\zeta - \dfrac{g}{V_{fd}} n_z \\ \dot{\sigma} = \zeta \\ \dot{\zeta} = -2\dfrac{V}{r}\zeta - \dfrac{g\cos(\varphi + k_V\sigma - \vartheta_0)n_z}{r} \end{cases} \tag{7}$$

III. SYNTHESIZE GUIDANCE LAW

Applying optimal control theory [4, 5, 6, 7], synthesizing guidance law with constrained conditions (5). Let us consider the cost function is as follows:

$$J = \frac{1}{2}\rho\left(\varphi\right)^2_{t_f} + \frac{1}{2}\int_{t_o}^{t_f} n_z^2 dt \to \min \tag{8}$$

The cost function has a quadratic form, in which: the first component ensures that the deviation at the time of setting is small; the second component ensures small flight device lateral acceleration. Where ρ are the semi-positive coefficients;

$$G = \frac{1}{2}\begin{bmatrix}\varphi\\ \sigma\\ \zeta\end{bmatrix}^T_{t_f} \left[P^f\right]_{3\times3} \begin{bmatrix}\varphi\\ \sigma\\ \zeta\end{bmatrix}_{t_f} ;\ P^f = \begin{bmatrix}\rho & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}.$$

The Hamiltonian function is as follows:

$$H = \lambda_\varphi\left(-\frac{g}{V_{fd}}n_z + \left(1-k_V\zeta\right)\right) + \lambda_\sigma\zeta + \lambda_\zeta\left(-2\frac{V}{r}\zeta - \frac{g}{r}\cos(\varphi + \mathrm{k}_V\,\sigma - \vartheta_0)n_z\right) + \frac{1}{2}n_z^2. \tag{9}$$

Where: $q = \vartheta_{fd} - \sigma;\ q_t = \vartheta_t - \sigma$. Co-state equations are defined as the following: $\dfrac{d\lambda_x}{dt} = -\dfrac{\partial H}{\partial x}$ (with co-state variables $x = \left[\varphi\ \sigma\ \zeta\right]^T$):

$$\begin{cases} \dfrac{d\lambda_\varphi}{dt} = -\lambda_\zeta \dfrac{g\sin(q)n_z}{r} \\ \dfrac{d\lambda_\sigma}{dt} = \lambda_\zeta k_V \dfrac{g\sin(q)n_z}{r} \\ \dfrac{d\lambda_\zeta}{dt} = -\lambda_\varphi\left(1-k_V\right) - \lambda_\sigma + 2\lambda_\zeta\dfrac{V}{r} \end{cases}. \tag{10}$$

The expression determines the optimal solution:

$$n_z = g\left(-\frac{1}{V_{fd}}\lambda_\varphi + \frac{\cos(q)}{r}\lambda_\zeta\right). \tag{11}$$

From Eq. (8), the Teminant function: $G = \dfrac{1}{2}\rho\left(\varphi\right)^2$, we have converted boundary conditions:

$$\begin{bmatrix}\lambda_\varphi\\ \lambda_\sigma\\ \lambda_\zeta\end{bmatrix}_{t_f} = \left[P^f\right]\begin{bmatrix}\varphi\\ \sigma\\ \zeta\end{bmatrix}_{t_f} ;P^f = \begin{bmatrix}\rho & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}. \tag{12}$$

P^f is a symmetric matrix. Eq. (7) can be rewritten in a matrix form:

$$\dot{x} = A^{(11)}x + Bu \,. \tag{13}$$

$$A^{(11)} = \begin{bmatrix} 0 & 0 & 1-k_V \\ 0 & 0 & 1 \\ 0 & 0 & -2\dfrac{V}{r} \end{bmatrix};\ B = \begin{bmatrix} -\dfrac{g}{V_{fd}} \\ 0 \\ -\dfrac{g}{r}\cos(q) \end{bmatrix};\ x = \begin{bmatrix} \varphi \\ \sigma \\ \zeta \end{bmatrix};\ u = n_z \,.$$

Co-state equations (10) are defined as the following in a matrix form:

$$\dot{\lambda} = A^{(22)}\lambda \,. \tag{14}$$

$$A^{(22)} = \begin{bmatrix} 0 & 0 & -\dfrac{g}{r}\sin(q)n_z \\ 0 & 0 & \dfrac{g}{r}k_V\sin(q)n_z \\ -(1-k_V) & -1 & 2\dfrac{V}{r} \end{bmatrix};\lambda = \begin{bmatrix} \lambda_\varphi \\ \lambda_\sigma \\ \lambda_\zeta \end{bmatrix}$$

From Eq. (11) and Eq. (13), we have:

$$\dot{x} = A^{(11)}x + A^{(12)}\lambda \,. \tag{15}$$

Where:

$$A^{(12)} = g\begin{bmatrix} -\dfrac{g}{V_{fd}} \\ 0 \\ -\dfrac{g}{r}\cos(q) \end{bmatrix}\begin{bmatrix} -\dfrac{1}{V_{fd}} & 0 & \dfrac{\cos(q)}{r} \end{bmatrix}.$$

And from Eq. (14) and Eq. (15) we obtain:

$$\begin{bmatrix} \dot{x} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} A^{(11)} & A^{(12)} \\ A^{(21)} & A^{(22)} \end{bmatrix}\begin{bmatrix} x \\ \lambda \end{bmatrix}. \tag{16}$$

Where: $A^{(21)} = [0]_{3\times3}$

Combining with boundary conditions (12), we have state variables and Co-state variables at t_f:

$$\begin{bmatrix} x \\ \lambda \end{bmatrix}_{t_f} = \begin{bmatrix} I_{3\times3} \\ P^f \end{bmatrix}x_{t_f} \,. \tag{17}$$

Considered at t, with $t_k \le t \le t_{k+1}$ and enough small, as $\sin(q) = \sin(q_k) = const;\ \cos(q) = \cos(q_k) = const$. Eq. (17) is considered linear. The solution of Eq. (16) is form:

$$\begin{bmatrix} x \\ \lambda \end{bmatrix} = \Phi(t_f - t)\begin{bmatrix} x \\ \lambda \end{bmatrix}_{t_f} \,. \tag{18}$$

Substituting Eq. (16) into Eq. (18), we get:

$$\begin{bmatrix} x \\ \lambda \end{bmatrix} = \begin{bmatrix} \Phi_{11}(t_f - t) & \Phi_{12}(t_f - t) \\ \Phi_{21}(t_f - t) & \Phi_{22}(t_f - t) \end{bmatrix}\begin{bmatrix} I_{3\times3} \\ P^f \end{bmatrix}x_{t_f} \,. \tag{19}$$

From Eq. (19), we have: $x = \left(\Phi_{11} + \Phi_{12}P^f\right)x_{t_f} \Rightarrow x_{t_f} = \left(\Phi_{11} + \Phi_{12}P^f\right)^{-1}x$

Similarly, we have: $\lambda = \left(\Phi_{21} + \Phi_{22}P^f\right)x_{t_f}$, Substituting x_{t_f} into the equation, we get:

$$\lambda = P^T(t)x \,. \tag{20}$$

From Eq. (12), (20), we get the boundary condition:

$$P^T(t_f) = P^f \,. \tag{21}$$

Two-sided derivative of Eq. (20), we get:

$$\dot{\lambda} = P^T(t)\dot{x} + \dot{P}^T(t)x \,. \tag{22}$$

From Eq. (14), Eq. (15), Eq. (21), Eq. (22) we get:

$$\dot{P}(t) = P(t)\left(A^{(22)}\right)^T - \left(A^{(11)}\right)^T P(t) - P(t)A^{(12)}P(t) \,. \tag{23}$$

From Eq. (11), Eq. (20) guidance law is:

$$n_z = g\begin{bmatrix} -\frac{1}{V_{fd}} & 0 & \frac{\cos(q_k)}{r_k} \end{bmatrix} P^T(t)x\,. \quad (24)$$

To find *P(t),* it is necessary to solve the Riccati equation (23) with boundary conditions Eq. (22). However, this is very difficult because Eq. (23) has no analytic solution. Therefore, here we find the approximate solution by choosing *P(t).* Choosing $P(t) = P^f$ as:

$$P = \begin{bmatrix} k & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \quad (25)$$

Replace Eq. (25) in Eq. (24) to get a guidance law:

$$n_z = -k\frac{g}{V_{fd}}\varphi\,. \quad (26)$$

Calculating the components on the right side of the Riccati equation (24), Eq.(26) gets:

$n_z = -\frac{V_{fd}}{g}\frac{1}{t+\frac{V_{fd}}{g}.\frac{\varphi_{max}}{n_{zmax}}}\varphi.$ Where: $t+\frac{V_{fd}}{g}.\frac{\varphi_{max}}{n_{zmax}} = t_{td} - t_g + \frac{V_{fd}}{g}.\frac{\varphi_{max}}{n_{zmax}} = t_0 - \frac{r}{V}.$ With, $V = -\dot{r},\ t_0 = t_{td} + \frac{V_{fd}}{g}.\frac{\varphi_{max}}{n_{zmax}};$ $t_g = \frac{r}{V}$ is the remaining flight time. The guidance law is determined by:

$$n_z = -\frac{V_{fd}}{g}\frac{1}{t_0 - \frac{r}{V}}\left(\Delta\psi - (1-k_V)\sigma\right). \quad (27)$$

The guidance law (27) is proportional to the coefficient of change depending on the FD-T distance and the approach speed. In the first stage, the value of the large coefficient has the effect of quickly putting the FD into a dynamic trajectory; the second stage has a small value to ensure that the FD remains stably in the trajectory.

IV. SIMULATION RESULTS

To simulate the evaluation of the efficiency of the conduction law (27), evaluate the parameters: Flight trajectory; Zero-effort miss; FD-T distance… The input parameters for the simulation evaluated in the paper are based on the data of the assumed FD.

Survey parameters: FD speed: $V_{fd} = 900m/s$; Lateral acceleration constraints: $n_{zmax} = 30$; Initial line-of-sight(LOS) angle: $\sigma_0 = -296,6^o$; Initial coordinates of MT: $\left(x_{t_0}, y_{t_0}\right) = (10,0)km$; T speed: $V_t = 20m/s$; Initial flight path angle of T: $\vartheta_t = 180^0$; The initial flight path angle of FD corresponds to 3 cases (FD1, FD2, FD3) $\vartheta_{fd0} = 355^0;\ 345^0;\ 335^0$, $q_0 = 58,4^0;\ 48,4^0;\ 38,4^0$.

Survey results:

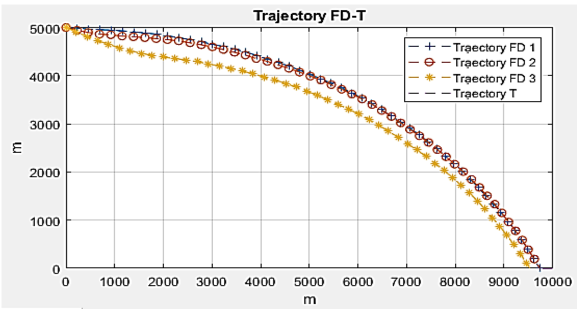

Figure 2. Trajectory FD-T

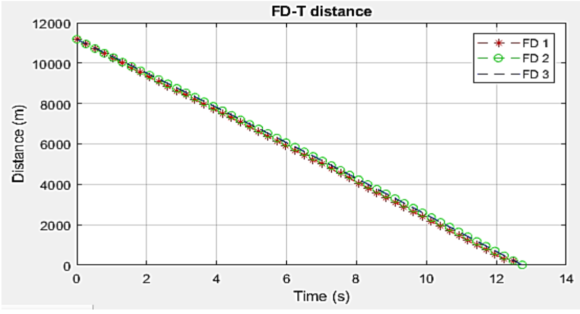

Figure 3. FD-T distance

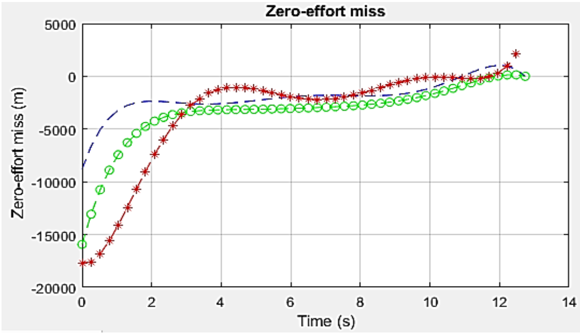

Figure 4. Zero-effort miss

Figure 2 shows the trajectories of FD and T in three different cases. Figure 3 is the distance between FD and T. Notice that in all 3 cases when following the guidance law (36) FD bring to T (Distance FD-T approaches "0"). Figure 4 shown the zero-effort miss of FD in 3 cases. With the look angle $q_0 = 48,4^0;\ 38,4^0$ the zero-effort miss is approaching "0", in case of large deviation the zero-effort miss is different from "0". This affects the FD's ability to break through T (the ground T is usually very solid, made of hard materials, so if the FD's orbital vector does not straight, it will affect the penetration ability even there. may be thrown out).

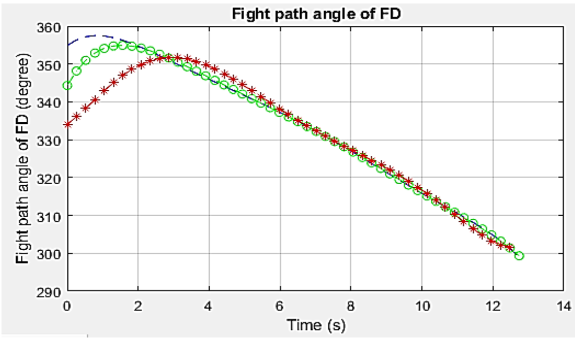

Figure 5. Fight path angle of FD

Figure 5 shows the flight path angle of FD through Figure 5; the conclusion about the guidance law is found to be consistent (36). The first stage creates a large overload that directs FD to T, then the FD overload decreases and tends to "0".

V. CONCLUSION

Through the survey results, it is found that the guidance law (35) ensures that the FD is brought to the fixed and mobile ground surface even in the case of a large initial deviation angle. However, in case the deflection angle is too large, the MT breaking efficiency is affected because the zero-effort miss at the time of encounter is different from "0".

The conduction law (35) is simple (only the device to determine the angle deviation and approach speed) can therefore be reduced to the device of the homing guidance, thus carrying more useful loads (explosives), which meets the combat requirements set out.

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