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Stability Analysis of the AA-10 Alamo Missile Control-Surface System Using a Fuzzy PID Control Method

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Abstract:*This paper presents the research results on designing a fuzzy PID controller for the electric actuator system of the P-27ET1 missile control surfaces. The proposed controller combines the robustness of the conventional PID control with the adaptability of fuzzy logic to improve system stability and response performance. Simulation results indicate that the fuzzy PID controller provides better dynamic characteristics and higher control precision compared to the classical PID controller. These findings form a foundation for further studies on the improvement and synthesis of the control loops for the AA-10 Alamo missile in particular, and long-range guided missiles in general.*

Keywords:*Fuzzy PID control, missile control surface, AA-10 Alamo, stability analysis, electric actuator, control loop synthesis..*

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I. INTRODUCTION

The process of receiving and processing information and generating control commands so that a missile tracks a target is highly complex. Onboard systems enable trajectory changes by applying systematic control actions to the missile; however, control is challenged by disturbances such as weather, temperature variations, terrain, and the operational environment. These factors significantly degrade the missile's controllability and reduce the accuracy of its terminal motion toward the target.

Therefore, the control problem requires effective filtering and mitigation of disturbance signals so that the missile rapidly reaches a stable state and can strike the target accurately. A common difficulty is that controllers designed to respond very quickly can induce high overloads, and when the missile is flying at high speed such overloads may jeopardize its structural integrity. To ensure maximal mission effectiveness, modern missiles combine optimal control techniques with advanced control strategies, including intelligent control methods.

With the objective of reducing oscillation time and increasing missile stability, this paper presents a mathematical model and investigates the stability of a fuzzy PID controller compared with a conventional PID controller. The study highlights the advantages and limitations of the modern control approach for actuating the P-27ET1 missile's control surfaces to achieve accurate target engagement.

II. MATHEMATICAL MODEL OF THE AA-10 ALAMO MISSILE

A propeller is a propulsion mechanism driven by a motor to generate rotational motion and produce thrust P. To determine the thrust generated by a rotating propeller that is moving axially at velocity V_0, consider a blade element at radius r with chord length band thickness dr (Figure 1). When the propeller rotates at angular velocity Ω, the tangential velocity of the blade element is $U_0 = r\Omega$. Applying the reverse motion principle, assume an airflow with axial velocity V_0 and tangential velocity U_0 acts on a stationary blade element. The resultant airflow velocity relative to the blade element is the vector sum $\vec{W}_0 = \vec{V}_0 + \vec{U}_0$ defined as the relative velocity.

2.1. Mathematical Model of the Tracking Coordinate System

To measure the control parameters in the AA-10 Alamo missile, a tracking coordinate system is used. The following set of equations describes the operation of this tracking coordinate system [4]:

$$\begin{cases} \varepsilon - \varepsilon_k = \Delta_k \\ T_k \dot{U}_k + U_k = K_k \Delta_k \\ T_y \dot{I}_y + I_y = K_y U_k \\ M_r = K_m I_y \\ \dot{\varepsilon}_k = \dfrac{M_r}{K_q} \end{cases} \qquad (1.1)$$

Where:
ε – the angle between the missile's longitudinal axis and the line of sight;
ε_k – the angle between the tracking coordinate system axis and the line of sight;
T_k – the time constant, with a value on the order of a fraction of a second;
U_k – the error signal;
K_k – the coefficient determining the dynamic characteristics of the tracking coordinate system;
K_γ – the power amplification coefficient of the tracking coordinate system;
K_m – the gyroscope frame moment coefficient;
Kq – the dynamic moment coefficient of the gyroscope drive;
M_r – the moment acting on the steering drive.
I_γ – the output signal of the tracking coordinate measurement device, which is an alternating current whose amplitude and phase contain information about the angular velocity and the position of the range vector.

In the case where the missile's inclination angles are zero, the position of the range vector is determined by the angle ε, and the position of the tracking-coordinate system axis is determined by the angle ε_k. The structural diagram of the tracking-coordinate device is thus:

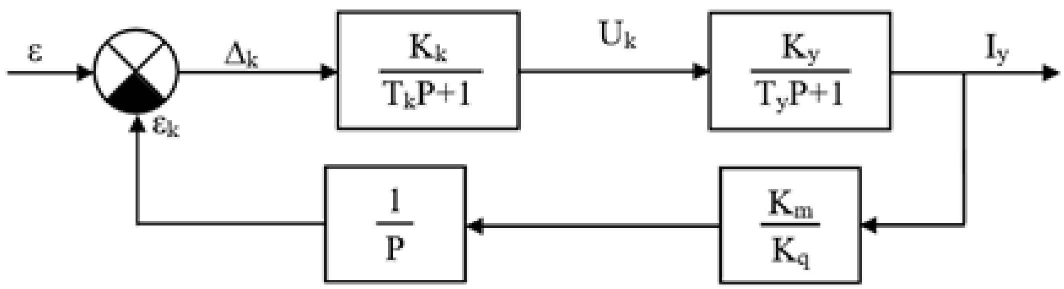

Figure 1. Structural diagram of the tracking coordinate system

Thus, in the tracking coordinate system, the output signal Iy is an alternating current proportional to the angular velocity of the relative range vector. The signal Iy serves as a control current, with its amplitude containing information about the value of the angular velocity of the relative range vector ω_D. To obtain the components of the vector, a reference signal with the same frequency as the alternating current Iy is required

2.2. Mathematical Model of the Steering Actuator

The steering actuator is used to transmit motion to the missile's control surfaces in accordance with the control signals from the infrared homing head.

The steering actuator includes two independent control channels, identical in both structure and characteristics. Here, we consider the electro-hydraulic actuator equipped with a servo-valve control system. The pressure source is generated by hydraulic fluid. The operation of the steering actuator is described by the following equations [3], [4]:

$$\dot{\delta} = k_{\pi}(\Delta M - M_{bl}) \quad (1.2)$$
$$\Delta M = k_{ml}\Delta I$$

In which:
δ – Deflection angle of the control surface;
M_{bl} – Hinge moment;

$$M_{bl} = M_{bl}^{\alpha}\alpha + M_{bl}^{\delta}\delta$$
$$M_{bl}^{\alpha} = m_{bl}^{\alpha}S_p l_p q \quad (1.3)$$
$$M_{bl}^{\delta} = m_{bl}^{\delta}S_p l_p q$$

Sp — Control-surface area;
lp — Control-surface chord;
q — Dynamic pressure, q=ρV2, where V is the missile speed;
$M_{bl}^{\delta}, M_{bl}^{\alpha}$ — Aerodynamic hinge-moment derivatives with respect to control-surface deflection δ and angle of attack α.
k_{ml} - Static transfer coefficient of the actuator;
k_{π} - Control-surface rate (velocity) coefficient.

According to the missile's operating principle, if the missile longitudinal axis is not aligned with the missile line-of-sight (range vector), the vertical channel will produce a deflection angle eke_kek in the homing/tracking coordinate unit. From this deflection signal, the control current at the output of the tracking coordinate unit is sent to the control-surface actuator; the actuator drives the control surface according to a given control law so as to steer the missile toward the target.

Thus, the block diagram describing the missile operation consists of two cascaded stages: the tracking coordinate unit and the control-surface actuator. The input is the control current (which carries the amplitude proportional to the angular deviation between the target velocity vector and the tracking-unit axis), and the output is the missile pitch rate θ.

III. INVESTIGATION OF THE SYSTEM WITH A FUZZY PID CONTROLLER

3.1. Steps for designing the fuzzy controller

To synthesize a fuzzy controller and make it fully operational, the following steps are performed:

Step 1: Analyze the plant and thereby define all input and output linguistic variables together with their domains of definition.

Step 2: Fuzzify the input/output linguistic variables.

Step 3: Build the control rules (composite propositions).

Step 4: Choose the inference/aggregation mechanism (e.g. MAX–MIN, MAX–PROD, SUM–MIN or SUM–PROD) and select the defuzzification method (mean of maxima, leftmost maximum, rightmost maximum, centroid, height).

Step 5: Defuzzify and optimize (tune) the controller.

3.2 Design and Implementation of the Fuzzy PID Controller

a. Development of a Fuzzy PID Controller for the Missile Control Surface System

Based on the theoretical framework presented above, the fuzzy proportional–integral–derivative (PID) controller was implemented in MATLAB/Simulink as follows:

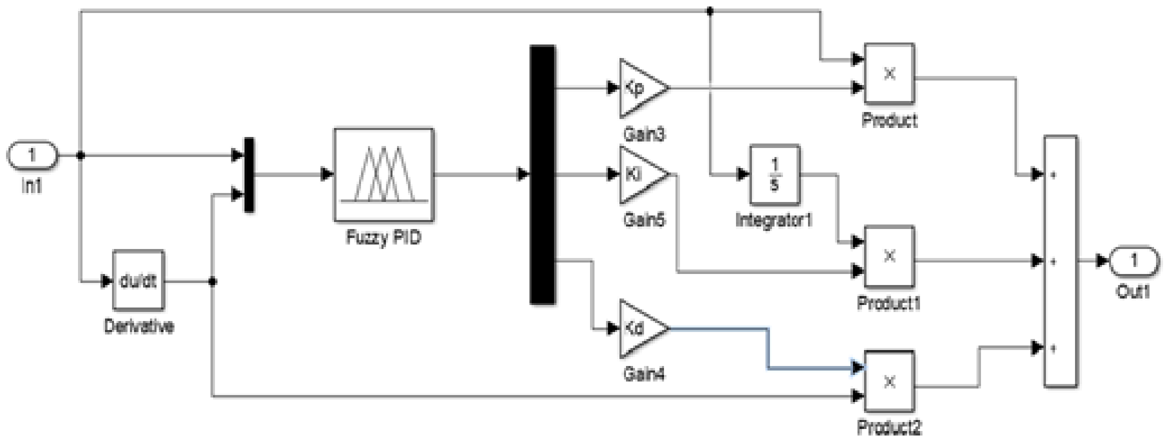

Figure 2. Block diagram of the fuzzy PID controller

b. Construction of Membership Functions and Rule Base

* Construction of the membership functions:

\- Determine the inputs / outputs:

Inputs (2): error ε— the angle between the missile longitudinal axis and the desired trajectory; and the derivative of the error ε˙.

Outputs (3): proportional gain Kp, integral gain Kiand derivative gain Kd.

\+ Define the linguistic variables:

Error: ET={negative large, negative medium, negative small, zero, positive small, positive medium, positive large}

Notation:ET={N3, N2, N1, ZE, P1, P2, P3}

Error derivative (DET):

DET={negative large, negative medium, negative small, zero, positive small, positive medium, positive large}

Notation:DET={N31, N21, N11, ZE1, P11, P21, P31}

\+ Controller outputs (linguistic terms):

Kp={zero, small, medium, large, very large}

Notation: Kp={Z, S, M, L, U}.

Ki={level 1, level 2, level 3, level 4, level 5}.

Notation: Ki={L1, L2, L3, L4, L5}.

Kd={zero, small, medium, large, very large}.

Notation: Kd={Z, S, M, L, U}K_d = \{Z,\,S,\,M,\,L,\,U\}Kd={Z,S,M,L,U}.

Construct the membership functions for the system: (to be specified — e.g., triangular/trapezoidal membership functions with explicit universes of discourse and breakpoints for each linguistic term).

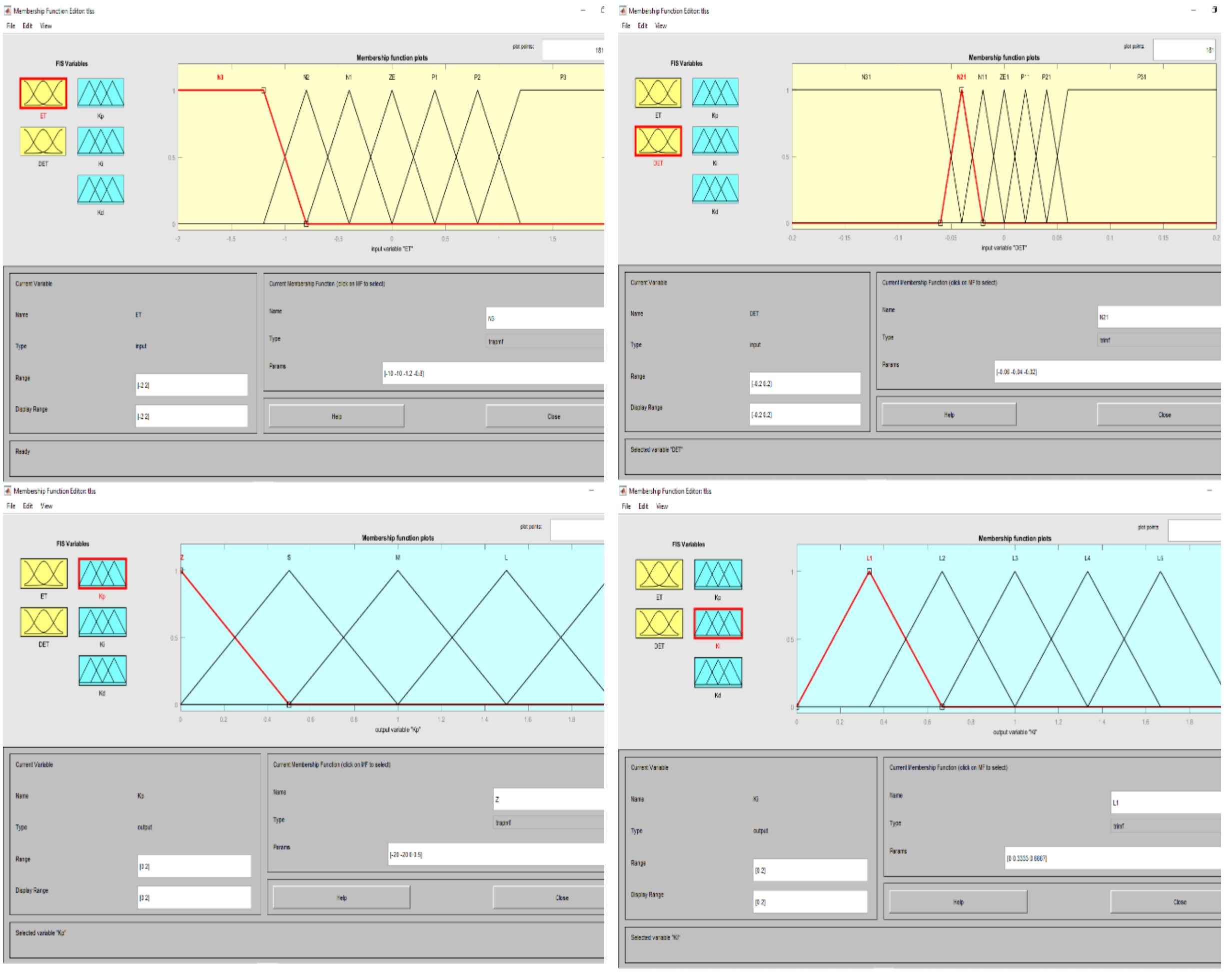

Figure 3:Construction of Membership Functions

Construction of the rule base:

- Kp control block:

Kp		DET						
		N31	**N21**	**N11**	**ZE1**	**P11**	**P21**	**P31**
ET	**N3**	U	U	L	M	S	Z	Z
	N2	L	L	L	L	L	L	L
	N1	M	M	M	M	M	M	M
	ZE	Z	Z	Z	Z	Z	Z	Z
	P1	M	M	M	M	M	M	M
	P2	L	L	L	L	L	L	L
	P3	U	U	L	M	S	Z	Z

- Kd control block:

Kd		DET						
		N31	**N21**	**N11**	**ZE1**	**P11**	**P21**	**P31**
ET	**N3**	U	U	L	M	S	Z	Z
	N2	L	L	M	M	M	L	L
	N1	M	M	M	M	M	M	M
	ZE	Z	Z	Z	Z	Z	Z	Z

	P1	M	M	M	M	M	M	M
	P2	L	L	M	M	M	L	L
	P3	U	U	L	M	S	Z	Z

- Ki control block:

Ki		DET						
		N31	**N21**	**N11**	**ZE1**	**P11**	**P21**	**P31**
ET	**N3**	L1	L1	L1	L1	L1	L1	L1
	N2	L3	L2	L2	L1	L2	L2	L3
	N1	L4	L3	L2	L1	L2	L3	L4
	ZE	L5	L4	L3	L2	L3	L4	L5
	P1	L4	L3	L2	L1	L2	L3	L4
	P2	L3	L2	L2	L1	L2	L2	L3
	P3	L1	L1	L1	L1	L1	L1	L1

3.3 Simulation Results and Comparison

The comparison results between the conventional PID controller and the fuzzy PID controller are presented in the table below. The system under investigation is the **actuator of the AA-10 Alamo** missile. The fuzzy PID controller, with three adjustable parameters Kp, Ki, and Kd, demonstrates superior performance — including a faster rise time, quicker response, and smaller overshoot — compared to the conventional PID controller.

An advantage of the fuzzy PID control approach is that it does not require an exact mathematical model of the system. Once the structural diagram is known, it becomes possible to design an optimal controller efficiently.

Table 1 presents the output response corresponding to the parameter set Kp=902.33, Ki=0.05, Kd=22.22, and ε=5as follows:

Table 1

Parameters	Overshoot		Settling time	
	fuzzy PID	conventional PID	fuzzy PID	conventional PID
Electric current Iy	4.2 %	37.1 %	0.19 s	11.7 s
Angle of attack α	18.1 %	38.1 %	1.5 s	11.71 s
Pitch angle θ	0.501 %	10.6 %	5.4 s	12.28 s

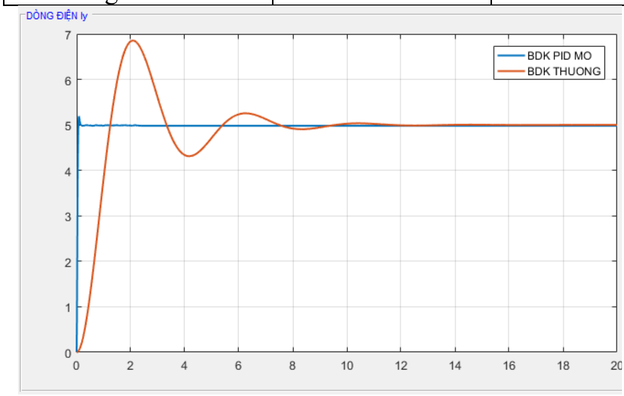

Figure 4. Comparison of the system current response between the fuzzy PID controller and the conventional controller, Kp=902.33, Ki=0.05, Kd=22.22 và ε = 5

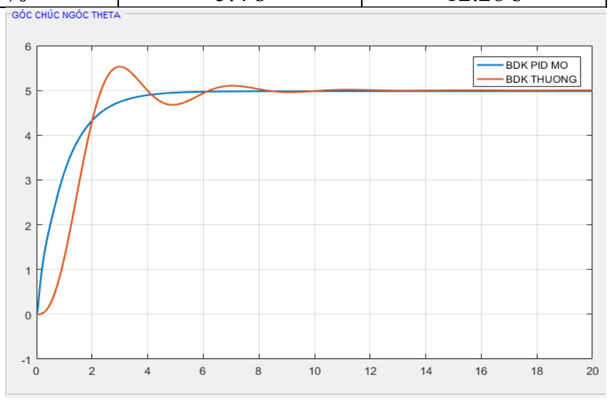

Figure 5. *Comparison of the system pitch angle response between the fuzzy PID controller and the conventional controller, Kp=902.33, Ki=0.05, Kd=22.22 và ε = 5*

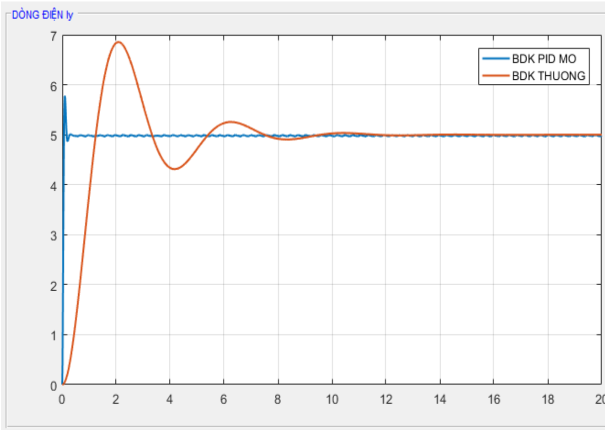

Figure 6. *Comparison of the system current response between the fuzzy PID controller and the conventional controller, Kp=500, Ki=0.5, Kd=12 và ε = 5*

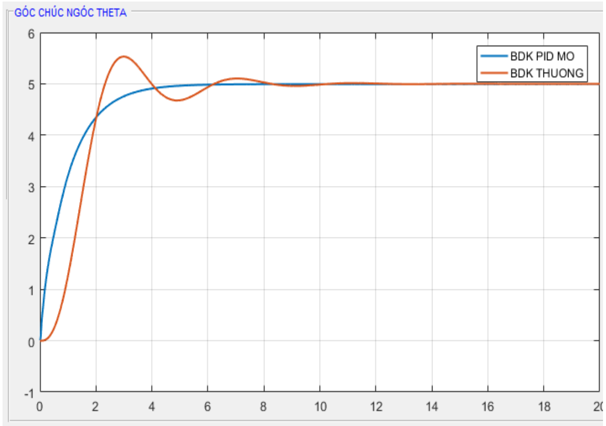

Figure 7. *Comparison of the system pitch angle response between the fuzzy PID controller and the conventional controller, Kp=500, Ki=0.5, Kd=12 và ε = 5*

Based on the above simulation results, it can be observed that when the missile fin electromechanical actuator is controlled using a fuzzy PID controller, the system exhibits *lower overshoot****,*** *faster transient response****,*** *shorter rise time, andsmaller steady-state error*, leading to an overall faster stabilization of the system.

Thus, the fuzzy PID controller effectively governs the missile's trajectory. However, the output current of the tracking coordinate unit still presents small oscillations, which in turn causes slight fluctuations in the angle-of-attack and pitch-angle control signals. These residual oscillations may affect the stability of the missile.

Therefore, for systems requiring high precision—such as missiles or aircraft—an additional current noise-filtering stage must be implemented at the output of the tracking coordinate unit to ensure optimal performance when using this controller.

IV. CONCLUSION

The paper investigates a fuzzy PID controller for the missile fin control system with the objective of reducing fin oscillations under the influence of noise and model uncertainties. Through MATLAB simulations and comparisons with a conventional controller, the results demonstrate that the proposed controller ensures superior performance, improved stability, and compliance with key missile performance criteria such as response speed, allowable overload, and target engagement time.

In addition to achieving the intended objectives, the study provides a foundation for further research on other missile control systems with similar characteristics. The obtained results also serve as a basis for the authors to develop a practical validation model, with the potential to replace the existing missile controller in future research.

Conflict of interest

There is no conflict to disclose.

FUTURE DEVELOPMENT

The next research direction is to develop a fuzzy PID controller integrated with artificial intelligence for automatic flight-control systems on missiles in particular, and for guidance/tracking control systems that require fast actuation and high precision in aerospace engineering in general.

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