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A review on Education Perspectives Research Conceptions

Dr. Dhinesha Ruwanthi Perera
Senior lecturer, Department of Management and Finance General Sir John Kotelawala Defence University, Sri Lanka
Dr.Anand K.Acharya
Prof. & Head, Department of Sociology Government Arts College, Bayad.
Dr.HareshD.Chaudhari
Prof. & Head, Department of History Government Arts College, Kwant.
Prof.BhavinS.Shah
Asso. Prof.Accountancy, Department ofAccountancy Gujarat Commerce College, Ahmedabad.
Dr.NayaneshA.Gadhavi
DepartmentofSociology
GujaratArts&ScienceCollege, Ahmedabad.
RakeshM.Patel
DepartmentofMathematics,GovernmentScienceCollege,Gandhinagar.
Equalcontribution:Allauthorsshareequalresponsibilityforthiswork.

Extended Abstract:
- *Mathematical modeling is a cyclical process in which mathematics is employed torepresent, explore, and better understand real-world phenomena.*
- *It involves mathematizing authentic situations—formulating problems, makingassumptions, defining quantities, applying mathematical tools, interpreting outcomes, and validating or revising results.*
- *Unlike traditional word problems, modeling tasks engage learners in creating and refining modelsratherthanapplyingpredeterminedprocedures.Mathematicalmodelinghas gained increasing recognition as a central component of mathematics education, underscored by frameworks such as the Common Core State Standards for Mathematics (CCSSM), particularly Mathematical Practice 4 (MP4)*
- *Model with mathematics. Modeling fosters the ability to connect mathematical reasoning with real-world contexts, supporting critical thinking, creativity, and conceptual understanding.*
- *This entry explores the evolving landscape of mathematical modeling education. It first reviews diverse conceptions and frameworks of modeling, highlighting distinctionsbetween holistic and atomistic approaches.*
- *Next, it examines the current state of research, including the documented benefits of modeling for student engagement and equitable access, alongside the challenges teachers face when implementing authentic modeling tasks.*
- *Finally, the entry discusses implications for future research and practice, emphasizing the need for targeted teacher preparation, curricular integration, and theoretical refinement to strengthen modeling within mathematics instruction.*

Keywords:
Mathematicalmodeling;modelingcompetencies;mathematization;mathematicseducation; real-worldapplications;modelingprocess;STEMintegration;problemsolving;modelingcycle; model validation; teaching and learning of modeling; curriculum development; teachereducation;authentictasks;educationalresearch;modelingpedagogy;modeling-based learning; mathematical practices; critical mathematics education; technology in modeling

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I. Literature review and introduction:

Mathematical modeling—often shortened to modeling—has become a prominent focus in contemporary mathematics education. At its core, modeling involves using mathematics as atool to interpret and analyze real-world phenomena, moving well beyond the scope of conventionaltextbookwordproblems.Modelingtasksinvitelearnerstograpplewith authentic situations by identifying assumptions, specifying relevant quantities, applying mathematical tools, and refining their solutions through interpretation and revision. The importance of this practice is underscored in policy and curriculum documents such as the Common Core State Standards for Mathematics (CCSSM), which position “Model with mathematics” (MP4) as a central mathematical practice [1]. This entry examines the growing body of scholarship on mathematical modeling education, with particular attention to developments in Western contexts. It begins by tracing different ways modeling has been conceptualized and the range of frameworks that describe how the process unfolds—from holistic, cyclical models to more fine-grained, step-oriented perspectives. The discussion then turns to empirical research, highlighting evidence of how modeling can support student engagement, conceptual understanding, and equitable access to mathematics, as well as the persistent challenges teachers face in implementing modeling tasks. The entry concludes with implications for future directions, calling for stronger teacher preparation, assessment design, and theoretical development to deepen the role of modeling in mathematics instruction. While this entry primarily attends to Western perspectives, it is important to acknowledge that mathematical modeling also has rich traditions in Eastern contexts, particularly in China and Japan.Acrossbothregions,modelingisvaluedforitspotentialtoconnectmathematicswith real-world problems, foster problem-solving competencies, and prepare students for STEM- related fields. However, notable differences emerge in emphasis and implementation. InWestern contexts, modeling is often framed as an open-ended process of inquiry, supporting creativity, collaboration, and sense-making. In contrast, in many Eastern contexts, it is positioned as a structured application of mathematics, more closely tied to canonical content, competitive examinations, or national curricular standards. These contrasts reflect broader cultural and educational traditions, yet both traditions share a common commitment toadvancingstudents’mathematicalreasoningandmodelingproficiency.Afullerexploration ofEasternperspectivesliesbeyondthescopeofthisentrybutremainsanimportantavenue for comparative research.

Defining Mathematical Modeling:

Mathematical modeling has gained substantial attention in recent decades as a centralcomponent of mathematics education. Despite this growing emphasis, there is no universally accepted definition of what modeling entails. Instead, a wide array of interpretations, perspectives, and frameworks populate the literature—each reflecting different goals, disciplinary traditions, and views on how mathematics interacts with the real world [2]. This conceptual diversity presents both opportunities and challenges for educators seeking tointegrate modeling into classroom practice. At its core, mathematical modeling begins with an authentic real-world situation that can be approached through multiple pathways and yield multiple viable solutions. A widely cited definition from the Guidelines for Assessment and InstructioninMathematicalModelingEducation(GAIMME)describesmodelingas“a process that uses mathematics to represent, analyze, make predictions, or otherwise provide insightintoreal-worldphenomena”[3].Thisdefinitionhighlightsthecyclicand iterative nature of modeling, where learners move between contextual understanding and mathematical abstraction—formulating a model, analyzing it, interpreting the results, and returning to the original context to evaluate validity and utility. A defining feature that distinguishes mathematical modeling from other forms of mathematical problem solving is the authenticity and complexity of the problems involved. Modeling tasks often stem from real-world contexts that are open-ended and ill-defined, lacking a single correct solution. These problems may involve ambiguous data or competing priorities, requiring students to simplify, clarify assumptions, and decide which variables and relationships are relevant. This necessity forsense-making, judgment, and justification makes modeling a powerful setting for developing mathematicalthinkingandreasoning.Studentsmustcoordinatebothmathematicalandreal-worldreasoningtoconstructmeaningfulanddefensiblesolutions.Theprocessoftranslating

betweenreal-worldphenomenaandmathematicalstructures—oftentermed mathematization—is central to modeling. This includes formulating relationships among variables, creating symbolic or graphical representations, and selecting appropriatemathematical tools (e.g., algebraic, statistical, computational). Once a model is developed and solved,studentsmustinterpretresultsincontext,determiningwhethertheirmodeloffers usefulorreliableinsights.Themodelingprocessisinherently iterative,ofteninvolving multiple rounds of refinement and validation to produce models that are both mathematically sound and contextually meaningful.

ModelingCompetencyasaFunctionofTime:

Let C(t) represent a student's modeling competency at time t. A simple learning curve could be expressed as:

$$\frac{dC}{dt}=k[C_{max}]-C(t) \tag{1}$$

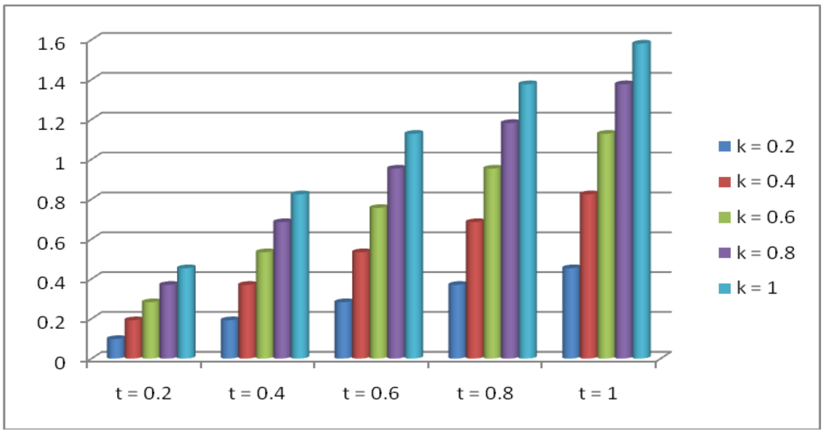
k=learningrate

C_{max}=maximumachievablecompetency

Solution:

$$C(t)=C_{max}\{1-e^{-kt}\} \tag{2}$$

Thismodelshowcompetencygrowsovertimeasstudentsgainexperience.

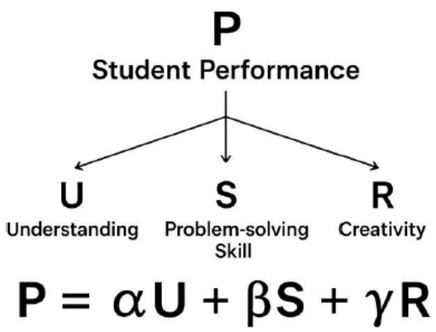


StudentPerformanceinModelingTasks:

Let P be student performance, influenced by understanding U, problem-solving skill S and creativity R:

$$P=\alpha U+\beta S+\gamma R \tag{3}$$

α, β, γ = weights reflecting contribution of each factor; this allows educators to analyze which skill contributes most to performance in modeling tasks.

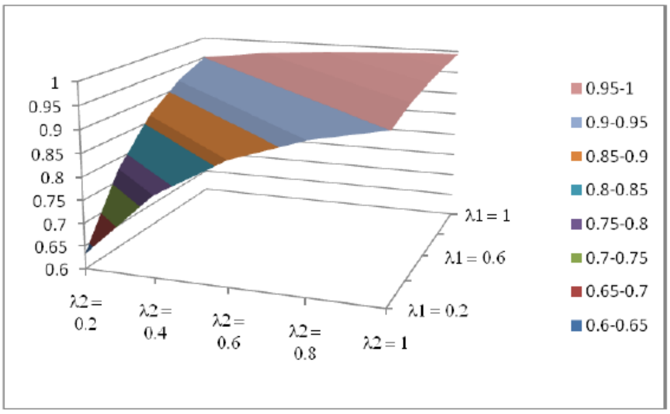


ProbabilityofSuccessfulModelFormulation:

Let p be the probability a student correctly formulates a mathematical model. If success depends on experience E and guidance G:

$$p=1-e^{-[\lambda_1 E+\lambda_2 G]} \tag{4}$$

λ_1, λ_2= sensitivity constants; this is a logistic-like growth function representing increasing likelihood with experience and guidance.



GraphicalSolution

ErrorinModelingProcess:

Let M_{real} be the real-world system and $M_{student}$ the student's model. The error can be quantified as:

$$\text{Error} = \|M_{real} - M_{student}\| \tag{5}$$

Using the L_2 norm (Euclidean distance) to measure deviation; this formalizes the accuracy of student-constructed models.

IterativeModelImprovement:

Let$M_n$representthemodelatiterationn.Improvementcouldfollow:

$$M_{n+1} = M_n + \eta(M_{target} - M_n) \tag{6}$$

η = learning/adaptation rate and M_{target}= ideal or real system model. This represents the iterative refinement process in modeling education.

1. DefinitionsandVariables

Let:

U(t) = comprehension/understanding of the problem at time t

F(t) = ability to formulate a mathematical model

S(t)=solution-findingandcomputationalskill

E(t) = evaluation/validation ability (how well students can assess their model)

R(t) = refinement capability (improving models based on feedback)

C(t)=overallmodelingcompetency G(t) =

guidance/support from teacherX(t) =

experience accumulated over time

k_i=learning/adaptationrateconstantsforeachstage

2. Stage1:Comprehension(UnderstandingtheProblem)

LearningisguidedbyexperienceX(t)andinstructionG(t):

$$\frac{dU}{dt} = k_1 X(t)\left[U_{max} - U(t)\right] + k_2 G(t) \tag{7}$$

U_{max}=maximumunderstandingachievable,Capturesgrowthinunderstandingwith experience and guidance

3. Stage2:ModelFormulation

ModelformulationdependsoncomprehensionU(t)andpreviousformulationskillF(t):

$$\frac{dF}{dt} = k_3 U(t)\left[F_{max} - F(t)\right] + k_4 X(t) \tag{8}$$

max=maximumformulationskillachievable,Showsthatbettercomprehensionimproves formulation ability

4. Stage3:Solution&ComputationalSkills

AbilitytosolvemodelsdependsonformulationF(t)andaccumulatedexperience:

$$\frac{dS}{dt} = k_5 F(t)\left[S_{max} - S(t)\right] + k_6 X(t) \tag{9}$$

S_{max}=maximumsolutionability

5. Stage4:Evaluation&Validation

EvaluationdependsonsolutionskillS(t)andcomprehensionU(t):

$$\frac{dE}{dt} = k_7 S(t)\left[E_{max} - E(t)\right] + k_8 U(t) \tag{10}$$

E_{max}=maximumevaluationskill

6. Stage5:ModelRefinement

RefinementusesevaluationE(t),solutionS(t)andguidanceG(t):

$$\frac{dR}{dt} = k_9 E(t)\left[R_{max} - R(t)\right] + k_{10} S(t) + k_{11} G(t) \tag{11}$$

R_{max}=maximumrefinementskill,Representsiterativeimprovement

7. AccumulatedExperience

Experiencegrowsasstudentspractice:

$$\frac{dX}{dt} = \lambda\left\{U(t)+F(t)+S(t)+E(t)+R(t)\right\} \tag{12}$$

λ=experienceaccumulationrate,Linksallstagestototalexperience

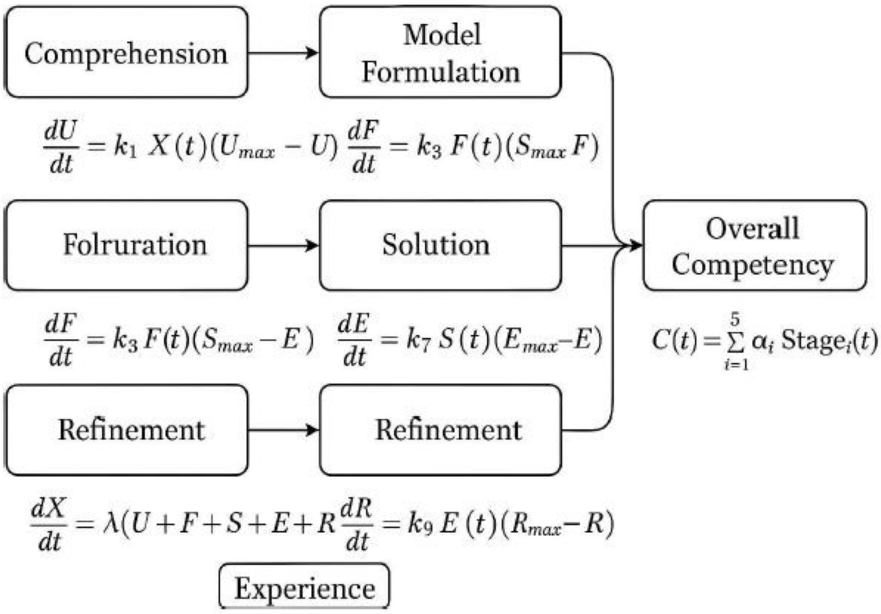
8. OverallCompetency

Finally,overallmodelingcompetencycanberepresentedasaweightedcombination:

$$C(t) = \left\{\alpha_1 U(t)+\alpha_2 F(t)+\alpha_3 S(t)+\alpha_4 E(t)+\alpha_5 R(t)\right\} \tag{13}$$

α_i=contributionweightsforeachstage

$$\begin{cases} \frac{dU}{dt} = k_1\, X(t)\,(U_{\max} - U) + k_2\, G(t) \\ \frac{dF}{dt} = k_3\, U(t)\,(F_{\max} - F) + k_4\, X(t) \\ \frac{dS}{dt} = k_5\, F(t)\,(S_{\max} - S) + k_6\, X(t) \\ \frac{dE}{dt} = k_7\, S(t)\,(E_{\max} - E) + k_8\, U(t) \\ \frac{dR}{dt} = k_9\, E(t)\,(R_{\max} - R) + k_{10}\, S(t) + k_{11}\, G(t) \\ \frac{dX}{dt} = \lambda(U + F + S + E + R) \\ C(t) = \sum_{i=1}^{5} \alpha_i\, \text{Stage}_i(t) \end{cases} \tag{14}$$



FrameworksfortheModelingProcess:

Various researchers have articulated distinct yet conceptually similar frameworks for the modeling process, offering valuable guidance for both instruction and assessment. Moreover, modeling frameworks may be designed with differing emphases—some primarily for research purposes, and others for practical or instructional application. One of the most influential frameworks is Blum's mathematical modeling cycle [4], illustrated in Figure 1. Designed primarily for research, this cycle underscores the distinction between the real world and the mathematicalworld.Theprocessbeginswhenamodelerencountersareal-worldsituation and identifies a problem tobe solved. To proceed,themodeler developsanunderstanding of thecontext,formingwhatistermedasituationmodel[5].Thisunderstandingisrefined

through the formulation ofsimplifying assumptions anddecisions, leading to the constructionof a real model—a structured representation of the problem that can be translated into mathematical terms. The next step, mathematization, transforms the real model into a mathematical model, which can then bemanipulated to yield mathematical results. Theseresultsaresubsequentlyinterpretedbackintothereal-worldcontexttoproducerealresults. The modeler must then determine whether these results adequately address the originalproblem. If they do not, the model is revised, and the process is repeated—a hallmark of the iterative nature of modeling represented in Blum's cyclic structure. The final stage, exposing, involves the communication or presentation of the model and its conclusions [4]. Arelated yet practice-oriented framework is presented in Bliss et al. [6], shown in Figure 2. Developed for practitioners and featured in a Society for Industrial and Applied Mathematics (SIAM) handbook,thiscyclesharesmanyfeatureswithBlum'smodel,includingitsiterative structure. However, Bliss and colleagues place greater emphasis on the early stages of the process—particularly researching the context, brainstorming approaches, and flexiblynavigating among problem definition, variable identification,and assumption formulation.Once a potential solution is developed, it is subjected to analysis and evaluation. Ifshortcomings are detected, the modeler returns to earlier phases for refinement. As in other frameworks, the process concludes with the communication of results. Not all modeling frameworks, however, depict a strict division between the real and mathematical worlds. For example, the semiotic-cognitive mathematical modeling cycle [7] acknowledges both realms while portraying them as interconnected rather than distinctly separated. In general, across numerous frameworks,themodeling process can be described inthe following sixstages [8]:

1. UnderstandingtheProblem–Identifyingandanalyzingthecontexttoformulatea clear problem statement.
2. Making Assumptions and Defining Variables – Simplifying the real-world situation by deciding which aspects to include or disregard.
3. FormulatingaMathematicalModel–Translatingthesimplifiedscenariointoa mathematical representation, such as equations, graphs, or systems.
4. Solving the Mathematical Model – Applying mathematical procedures and techniquesto derive results.
5. InterpretingandValidatingResults–Relatingmathematicaloutcomesbacktothe real-world context and assessing the model's adequacy and accuracy.
6. ReportingFindings–Communicatingthemodel'sresults,implications,limitations, and possible improvements.

Crucially,researchindicatesthatthesestagesarenotstrictlylinear.Modelersoftenmove back and forth between steps, revisiting assumptions or redefining variables as new insights emerge[9].Furthermore,validation,acriticalyetsometimesunderemphasizedcomponent, can occur at multiple stages of the modeling process rather than solely at the end.

ModelingCompetenciesandHolisticandAtomisticApproachestoModeling:

Maaß [10] identified a set of core modeling competencies that include: understanding the problem, setting up a mathematical model, solving the model, interpreting results, andvalidating the solution—each of which can be further divided into sub-competencies. For instance, understanding the problem may involve identifying relevant quantities and making appropriate assumptions, while formulating a model requires choosing suitable mathematical tools and representations. Maaß's framework is one of the most frequently cited in theliterature, though other conceptualizations and distinctions among modeling researchers also exist [11]. Building on this foundation, Niss and Højgaard [12] distinguished between general andspecificdefinitionsofmathematicalcompetence.Generaldefinitionsadoptaholistic view,describingcompetenceasanintegratedcapacitytousemathematicsmeaningfully across contexts. Specific definitions, by contrast, take an atomistic perspective, decomposing competenceintodiscrete,teachable,andassessablesub-competencies.Thisdistinction parallelstheholistic–atomisticapproachesinmathematicalmodeling:holistictasks correspond to the general view of competence, while atomistic tasks align with the specificview. Given the complexity of the modeling process, educators have developed various instructional approaches to support student engagement, broadly categorized as holistic or atomistic [13]. A holistic approach engages students in the entire modeling cycle from start to finish [13]. Students confront an authenticreal-world problem, make assumptions,construct and solve mathematical models, interpret their results, and refine their solutions throughiterative cycles. Such tasks are typically open-ended, allowing for multiple solution paths and diverse representations. The holistic approach emphasizes the integration of all modeling competencies, helping students appreciate how mathematical thinking operates within real contexts. In this way, holistic tasks embody the general view of competence—an overarching, integratedabilitytoapplymathematicsmeaningfully.Anexampleofaholisticmathematical modelingtaskistheChernobyldisastertask[14].Inthisactivity,studentswatchan informational video about the Chernobyl disaster and are asked to imagine being recruited bytheNationalRegulatoryCommissiontodeterminewhichradioactivesubstancereleased during the event had the greatest impact. The phrase "greatest impact" is intentionally open to interpretation, making the task inherently open-ended and conducive to multiple realistic solutions. Students receive a spreadsheet containing data on the radioactive substances,includingtheirhalf-lives,initialquantitiesreleased,andsafeexposurelimits.Theymust define what is meant by "greatest impact," then engage in mathematical reasoning to compute theremainingquantityofeachsubstancetodayandestimatewhenitssafeexposurelimit wouldbereached.Bycomparingsubstancesandconsideringbroadereffectsonpeopleand the environment, students iteratively apply modeling steps to justify their conclusions about which substance had the most significant impact. Another exampleof aholisticmodeling taskis the shoebox filled with cash problem [15]. Here, students must determine whether a witness who claimed to see a man drop a shoebox containing USD 70,000 in small bills is telling the truth. Because the problem is open-ended and requires engagement with all phases of the modeling cycle—from problem analysis to assumption-making, computation, andjustification—itexemplifiesaholisticmodelingtask.Incontrast,anatomisticapproach focuses on specific components or sub-competencies within the modeling process [13].Students might, for example, practice only formulating assumptions, interpreting results, or validating a given model.

Atomistic tasks are especially valuable for scaffolding learning, enablingstudentstobuildfoundationalmodelingskillsinisolationbeforecoordinatingthem in full modeling activities. Although they may lack the complexity and richness of complete modeling cycles, such tasks support the gradual development of modeling proficiency. This reflects the specific view of competence, where individual sub-skills are taught andstrengthened separately before being integrated. An example of an atomistic modeling task isthe long jump task [16], adapted from the College and Career Readiness Sample Mathematics Tasks[17]andshowninFigure3.Studentsaregivenlongjumpdistancesforthreeathletes and asked to justify which athlete should advance to the championship. Constructing these justifications helps students practice making assumptions. For instance, if Elsa is chosen, one possible assumption might be that the athlete with the single longest jump should qualify. Through these discussions, students explore the role of assumptions without engaging in the complete mathematical modeling cycle. Although the task allows for open-ended reasoning, it does not require full model creation or quantitative computation—hence, it is considered atomistic.Anotherexampleofanatomistictaskinvolvescomparingpricingmodelsacross fourcandystores.Studentsreceivefourdistinctrepresentations—suchasatable,agraph,a writtendescription,andanalgebraicrelationship—andareaskedtoevaluatewhichstore offers the best deal [13]. This task focuses specifically on interpreting and contrasting models, not on model creation, and therefore fits within the atomistic framework. Both holistic and atomistic approaches offer significant pedagogical value, and effective modeling instruction oftenbalancesthetwoaccordingtolearners’developmentalstagesandinstructional objectives.Similarly,NissandHøjgaard[12]arguethatbothgeneralandspecificdefinitions of competence are essential and complementary: the general view articulates a vision of comprehensive mathematical capability, while the specific view operationalizes thiscompetence for teaching, assessment, and curriculum design.

To support theintegration ofmodeling in mathematics classrooms, future work must continuetoinvestigatehowteacherslearntodesign,implement,andreflectonmodelingtasks. Research should explorehowteacher beliefs evolvethroughpracticeand identify structures that support this evolution. As mathematical modeling becomes more central to mathematics curricula, it is essential that teachers are equipped not only with technical tools but also with pedagogical mindsets that embrace complexity, uncertainty, and student-centered inquiry.

ImplicationsforFutureResearchandPractice:

Research on mathematical modeling in education has grown significantly over the past two decades, yet important questions remain regarding how to prepare teachers, structure learning environments, and refine theoretical frameworks that guide instruction and research. Mathematics education organizations such as the Association of Mathematics TeacherEducators (AMTE), the National Council of Teachers of Mathematics (NCTM), andPsychologyofMathematicsEducation(PME)continuetopromotemodelingthrough research, working groups, position statements, workshops, and teacher resources [47–51]. The benefitsofmathematicalmodeling—increasedengagement,accesstomeaningful mathematics, and opportunities for creative and critical reasoning—are well-documented. However,thesebenefitsarenotautomatic;theydependheavilyonthedecisionsteachers make and the kinds of learning environments they create. The field must deepen its understanding of the professional knowledge,beliefs, and instructional moves that enablerobust modeling experiences in classrooms.

PreparingTeachersfortheComplexitiesofMathematicalModeling:

One clear implication of the literature is that teachers need sustained, targeted preparation to integrate modeling effectively [30,41]. Unlike conventional problem solving, mathematical modeling involves open-ended, context-rich tasks that require students to define problems,make assumptions, construct representations, and revise solutions based on real-world constraints.Facilitatingsuchactivitiesdemandsateachingorientationthatembraces ambiguity,valuesstudentthinking,andsupportsdiscourseacrossdiversemathematical

pathways.Manyteachers,especiallythosewithoutpriormodelingexperience,struggleto enact this vision. Research consistently shows that teachers may hold limited or incorrect understandings of modeling [33]. Some conflate it with using physical representations (e.g., base-tenblocks),while othersviewitasa linear,stepwiseprocess,reducing theexploratory and authentic character of modeling tasks. Professional development must go beyond task introductiontohelpteachersdevelopconceptualunderstandingofmodelingasa mathematicalprocessandbuildapedagogicalrepertoireforstudent-centeredinquiry. Programs such as the LEMA project [40] and the four-phase model by Tan and Ang [41] illustrate how structured, iterative PD can improve teachers' confidence and effectiveness. However, as Alhammouri & DiNapoli [31] and Taite et al. [43] note, even well-designed PD cannotresolvealltensions;teachersmaystillexperiencediscomfortorinternalconflictas they reconcile modeling with prior beliefs about mathematics and pedagogy. Future teacher preparation must attend to these affective and epistemological dimensions [49, 52]. Teachers needopportunitiestointerrogateassumptionsaboutwhatcountsasmathematicalreasoning and whose knowledge is valued. PD should incorporate structured reflection on instructional strategies, teacher beliefs, positionalities, and classroom norms [53]. Reflection should be situatedinrichpedagogicalexperiences,suchasanalyzingstudentwork,co-teaching modeling lessons, or engaging in collaborative design cycles. Participation in modeling as learners themselves can be transformative, shifting teachers' perceptions of mathematics and student agency. Longitudinal research is needed to investigate how various PD models— coaching, lesson study, co-teaching, action research—support shifts in teacher practice and perspective over time, particularly regarding conceptual and pedagogical growth.

AdvancingTheoreticalFrameworksforMathematicalModelingEducation:

In addition to supporting teachers, the field requires continued theoretical refinement. A major challengeis thewidevariability inconceptualizationsofmodeling.Definitionsrange from task-oriented approaches, emphasizing real-world context and open-endedness, to process- oriented views, highlighting cycles of problem formulation, mathematization, analysis, and revision. This inconsistency complicates the comparison of studies and the development of generalizableinsights.Theoreticalframeworkscanhelpbring coherence.Forinstance, Blomhøj and Jensen [54]'s modeling cycle offers a structure for understanding student and teacherengagementinmodelingactivities.However,manyframeworksassumeacertain levelofmathematicalandpedagogicalfluencyandmaynotfullyaccountfortherangeof

teacher moves or student experiences in diverse classrooms, especially those with varying linguistic,cultural,andsocioeconomicresources.ResearchbyCirilloetal.[23,25] emphasizes the importance of considering equity and access. Modeling can be culturally responsive when students are encouraged to draw on knowledge from their homes and communities, making it a site for identity development and sociopolitical learning. Future theoretical work should explore how modeling supports justice-oriented goals, includingstudent empowerment, critical reasoning, and real-world problem solving grounded in community concerns.

Frameworks for Classroom Discourse and Teacher Questioning in Mathematical Modeling:

Effective facilitation of mathematical modeling tasks relies on strong classroom discourse frameworks.Forexample,SmithandStein[37]'sfivepracticesfororchestrating mathematical discussions— anticipating, monitoring, selecting, sequencing, and connecting— can be adapted and extended to modeling contexts, supporting teachers in managing complex studentinteractionsandreasoning.Furtherinsightsintofacilitationstrategiescomefrom Zhangetal.[38],whohighlightmethodsforsupportingcollaborativeproblemsolvingin smallgroups,emphasizingteachermovesthatfostercommunity,sustaindialogue,and promote metacognitive awareness. These frameworks provide both language and structure for describingthecomplexinterpersonalandintellectualworkteachersengageinwhile facilitating modeling. Teacher questioning strategies are also critical in supporting student thinkingandlearningduringmodelingactivities.Taite[16]proposedapreliminary questioning framework specific to modeling, building on general classroom questioning types from Boaler and Brodie [55]. Observations of teacher questioning during student modeling presentations revealed four main categories:

1. Sensemaking–Questionsthathelpstudentsclarify,interpret,orjustifytheir reasoning.
2. Operational–Questionsrelatedtothesteps,procedures,ormathematicaloperations used in the model.
3. Higher-order thinking – Questions that encourage analysis, synthesis, or evaluation ofthe model and its assumptions.
4. Increasing validity – Questions that prompt students to verify results, test assumptions,or critically reflect on their modeling process.

Somequestioningtypes,suchasgatheringinformationandgeneratingdiscussion,are described by Boaler and Brodie, while others, such as restating and nature of modeling, aremore specific to modeling as identified by Taite [16]. These questioning categories warrant further exploration as a means to help teachers expand their repertoire of questions, ultimately strengthening students' modeling solutions and deepening mathematical reasoning (see Table4).

DocumentingTeacherFacilitationandClassroomDynamics:

To support theory building, future research should document and analyze the nuanced ways teachers facilitate modeling discussions, including how they:

- Positionstudents,
- Manageuncertainty,and
- Navigatecompetinginstructionalgoals.

Fine-grained analyses of teacher talk, student interactions, and classroom artifacts canilluminate the micro-level decisions that shape modeling experiences. Moreover, attention to affective dynamics—such as frustration, curiosity, and satisfaction—can enrich understandingof how students and teachers experience modeling as both cognitively and emotionallyengaging work.

EmbracingComplexityandSupportingMeaningfulIntegration:

Efforts to expand modeling in mathematics education must embrace its inherent complexity. Modeling cannot be reduced to a checklist of best practices or a standardized curriculum supplement; it is a dynamic, multifaceted practice that challenges conventional norms of teaching and learning. Successful integration requires alignment of multiple factors: teacher beliefs and knowledge, curriculum design, assessment practices, classroom culture, and institutional supports. Research demonstrates that modeling can broaden participation in mathematics, creating space for diverse ways of knowing and doing. Students engaged in modeling have opportunities to formulate questions, make decisions, and justify reasoning, activitiesoftenmarginalizedinproceduralapproachestomathematics.However,this potentialisonlyrealizedwhenteachersarepreparedtosupportstudentagencyandnavigate theambiguityandopennessinherentinmodelingtasks.Consequently,teacherlearningmust

remain central to modeling reform. This includes pre-service education as well as ongoing, embedded professional development that allow teachers to experiment, reflect, and grow. Teacher education programs should expose candidates to modeling early and often, providing multiple opportunities to engage as learners, observe modeling in practice, and develop the facilitation skills needed to support diverse learners.

RefiningTheoryandResearchPractices:

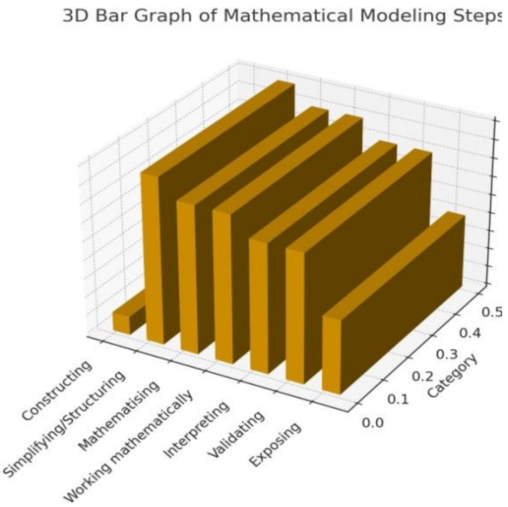
Future research should continue to refine the conceptualization, enactment, and study of modeling. Key directions include:

- Expandingthetheoreticalvocabularyformodeling,
- Examiningthesocialandculturaldimensionsofmodeling,and
- Developinganalyticaltoolstounderstandcomplexteachingandlearninginteractions.

Comparative and cross-national studies could provide valuable insights into how different educationalsystemsapproachmodeling,identifycommonchallenges,andhighlight innovative practices.

ConcludingThoughtsonFutureImplications:

Mathematical modeling sits at the intersection of mathematics,pedagogy, and the real world.As a practice, it invites students to engage deeply with mathematical ideas while addressing meaningful,contextuallyrelevantproblems.Asaninstructionalapproach,itchallenges teachers to reimagine their roles, trust in student reasoning, and embrace complexity and uncertainty. The current research highlights both the promise and challenges of modeling in mathematics education. Moving forward, the field must invest in sustained, reflective, and theoretically grounded approaches to teacher preparation and professional development. Only through such investment can modeling evolve from the margins of mathematics instruction to become a central, equitable, and transformative component of classroom practice.



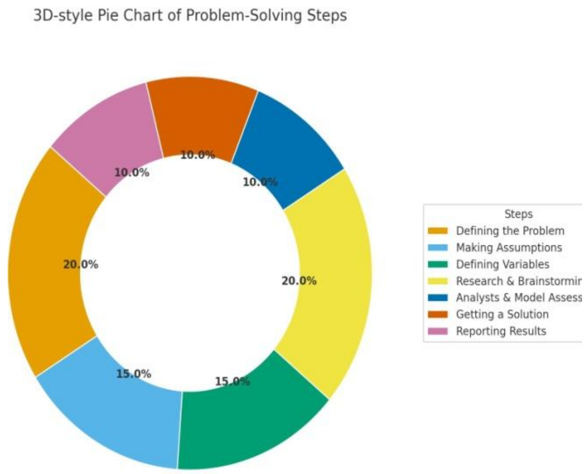


Figure 1. 3D bar graph representing the seven mathematical modeling steps (Constructing, Structuring, Mathematising, etc.). Blum's mathematical modeling cycle. Reprinted with permission from ref. [4]. Copyright 2011 Springer Science + Business Media B.V.

Figure 2. 3D-style pie chart (donut chart) showing the problem-solving steps from the diagram. Bliss et al.'s mathematical modeling cycle. Reprinted with permission from ref. [6].
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Priya	Riya	Ananya
1.45	1.55	1.5
1.52	1.57	1.62
1.6	1.53	1.59
1.48	1.61	1.54
1.55	1.58	1.63
1.5	1.6	1.61

Athlete	AverageJump	BestJump	Consistency(StdDev)
Priya	1.52m	1.60m	0.05
Riya	1.57m	1.61m	0.03(mostconsistent)
Ananya	1.58m	1.63m	0.05

Figure3.TheLongJumptask.Reprintedfrom[16](p.24).

Benefits and challenges with Mathematicnodling

Benefits	Challenges
• Accessible and engaging • Can build on students' interests • Opportunity for both mathematical and real-world connections • Opportunity for productive struggle	• Teachers' conceptions of mathematical modeling • Implementing the modeling process • Anticipating multiple student approaches anol perspectives • Finding time to implement tasks

Table 1. Modeling mathematics vs. mathematical modeling.

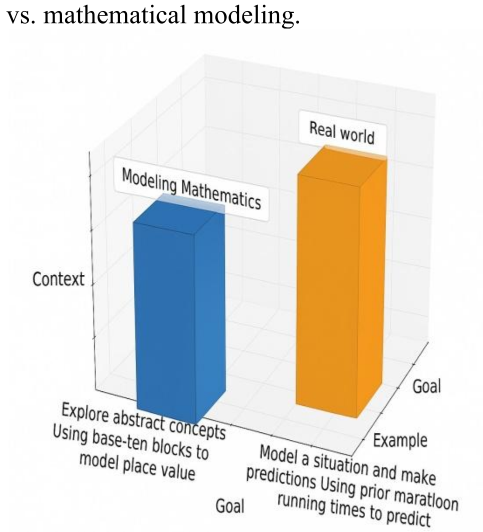


Table 2. Benefits and Challenges of Modeling mathematics

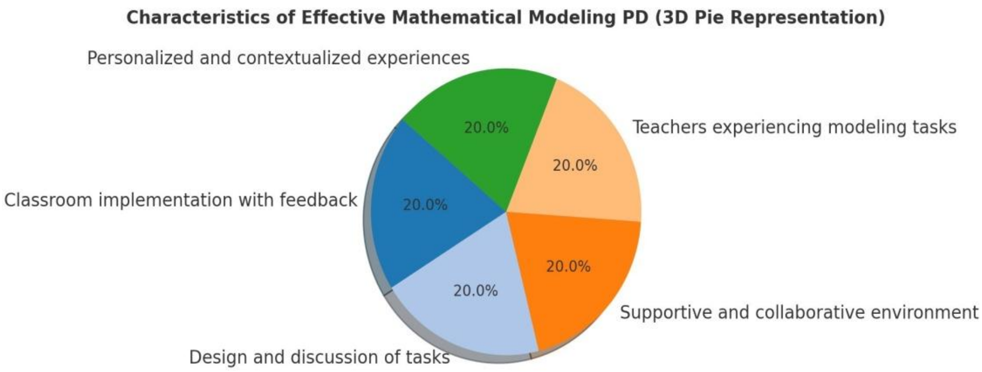


Table 3. **3D-style pie chart** representing the Characteristics of Effective Mathematical Modeling Professional Development (PD), with equal weighting for each characteristic.

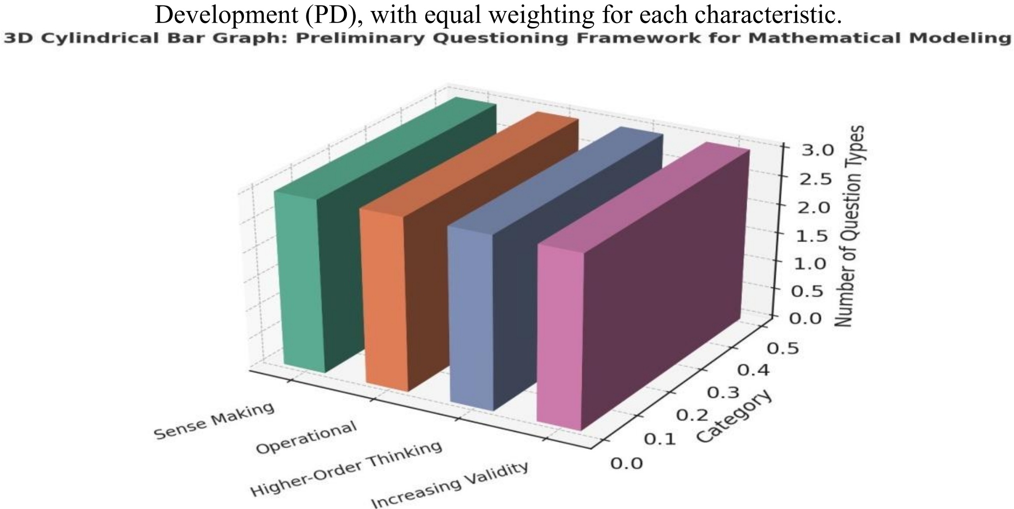
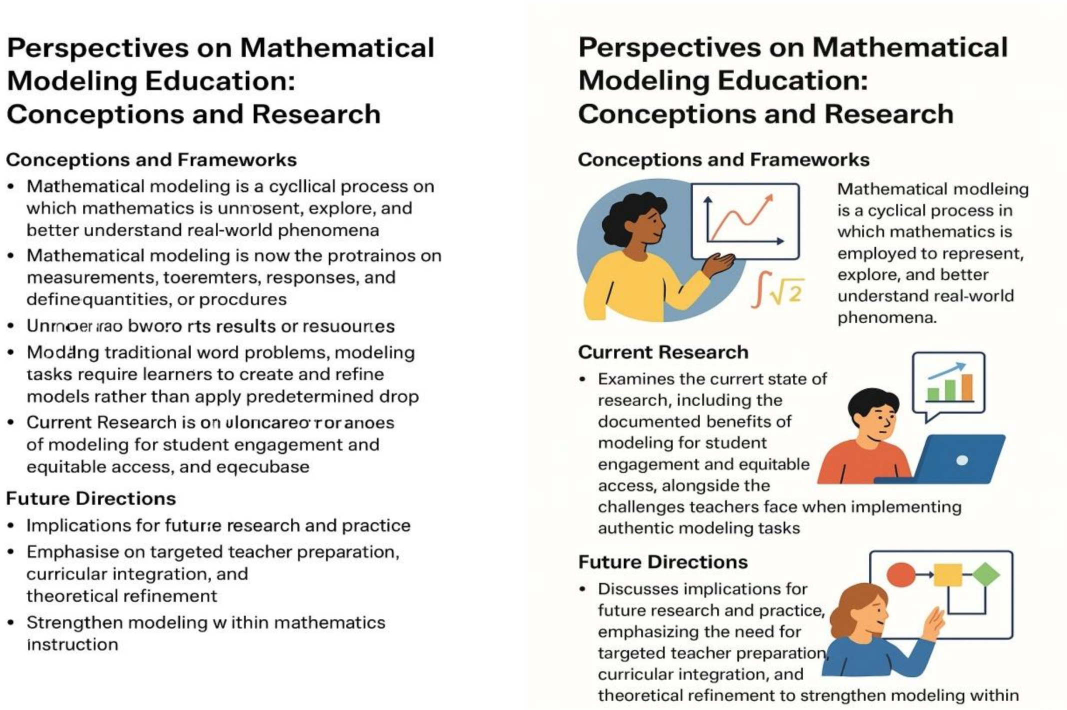


Table 4. **3D cylindrical bar graph** visualizing the Preliminary Questioning Framework for Mathematical Modeling, with each category (Sense Making, Operational, Higher-Order Thinking, Increasing Validity) represented by cylindrical bars showing their respective question type counts.

GraphicalAbstract



Declarations:
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ConflictofInterest: Theauthorsdeclarethattheyhavenoknowncompetingfinancial interests or personal relationships that could have appeared to influence the work reported inthis paper.
Data Availability Statement: The data used in this study were obtained from publiclyavailable online sources.
InstitutionalReviewBoardStatement: Notapplicable.
InformedConsentStatement: Notapplicable.
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