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A Method of Fuzzy-LQR Algorithm for Controlling Self-Balancing Bicycle using Reaction Wheel

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Abstract: *The bicycle is a popular means of transportation. However, the bicycle is typically an unstable system, it will fall down without the assistance of rider like steering the handle or moving their upper body. Every rider needs to practice to ride a bicycle. Therefore, it is essential to develop a balanced support system for the bicycle rider. This paper presents the balance control of the bicycle with the reaction wheel. A simple model of the bicycle is generated by the Euler-Lagrange method. A Fuzzy-LQR controller is designed and compared to a popular LQR controller. The Fuzzy-LQR controller is proved to be better than the LQR controller through Matlab/Simulink simulation.*

Keywords: *Bicycle, Fuzzy-LQR, reaction wheel, Euler-Lagrange.*

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I. INTRODUCTION

These days, people mostly use cars or motorbikes as their main vehicles because of their convenience. Besides, bicycle is also a good choice of many people due to environmental friendliness and health benefits that it brings. For many people, cycling can be quite simple. However, there are still people who meet a lot of difficulties, especially young children, adults who never ride a bicycle, people who are injured, people who have problems with cognitive development. A system that is capable of supporting the balance of bicyclists can provide great benefit to this group of people. The system can be used as a training tool for cycling or a device that supports physical therapy.

The main idea of self-balancing bicycle is using sensors to measure the angle of bicycle against the vertical and use the actuators to bring it to a balanced position. Based on this idea, four types of self-balancing bicycle were researched and introduced [11]. The first is the balance based on the center of gravity or the "mass balancer" [1]. This type of bicycle has a simple mechanical structure but the torque it can produce is small. The second is steering-control [2] where a controller control the amount of torque applied to the steering handlebar to balance the bicycle. The third is gyroscopic stabilization [3], this method can produce a large torque, but the energy consumption is very high because the flywheel is spinning all the time. Besides, its complex structure is also a disadvantage. The fourth type is balancing control using reaction wheel [4]. In this method, the motor which is attached to reaction wheel has the spin axis parallel to the bicycle. When the bicycle falls to one side, the motor will cause the reaction wheel to rotate, generating the reactionary torque, which brings the bicycle back to the balanced position. Disadvantages of this method are high energy consumption and producing small torque. However, it is low cost and the mechanical structure is simple.

This article uses the fourth model for self-balancing bicycle. There have been some studies that use reaction wheel to balance the bicycle [4-11]. In which, PID and LQR are algorithms that commonly used. In this article, based on the successful results of the studies [6], [10], [11], the author proposed using the Fuzzy-LQR algorithm, this algorithm has also been used for research [12-14].

II. MODEL OF SELF-BALANCING BICYCLE

To control the balance of bicycle, we assume that the whole bicycle is a reaction wheel pendulum as in Fig. 1. The system's parameters in Fig 1 are shown in the table below [11]:

Symbols	Parameters	Values
O	The origin coordinates of the system	
O_1	The center of mass of the bicycle	

O_2	The center of mass of the reaction wheel	
θ	The angle between the bicycle and the vertical upright direction	
ϕ	The rotation angle of reaction wheel	
g	Gravitational acceleration	9.81 m/s^2
L_1	The distance from the origin O to the center of mass of the bicycle	0.25 m
L_2	The distance from the origin O to the center of mass of the reaction wheel	0.35 m
m_1	The bicycle mass	20.1 kg
m_2	The reaction wheel mass	3.7 kg
I_1	The bicycle moment of inertia about its center of mass	1.504 kgm^2
I_2	The reaction wheel moment of inertia about its center of mass	0.052 kgm^2
R_m	The armature coil inductance	0.71 Ω
K_t	The motor torque constant	0.0229 Nm/A
K_e	The motor back emf constant	0.0229 Vs/rad
N_g	The motor gear ratio	25:1

Table 1. The parameters of the system

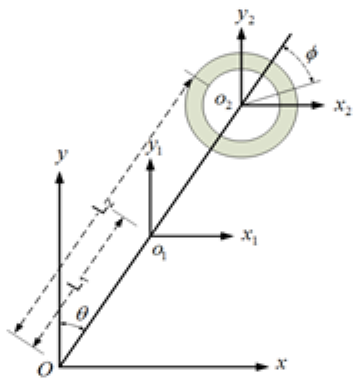


Figure 1. The model of self-balancing bicycle using reaction wheel [11]

In this paper, the mathematical equations are obtained by Euler-Lagrange method. The total kinematic energy of the system can be expressed as follow:

$$T = \frac{1}{2}(m_1L_1^2 + m_2L_2^2 + I_1 + I_2)\dot{\theta}^2 + I_2\dot{\theta}\dot{\phi} + \frac{1}{2}I_2\dot{\phi}^2 \tag{1}$$

The potential energy of the system is obtained as below:

$$U = (m_1L_1 + m_2L_2)g\cos\theta \tag{2}$$

Then, the Lagrange operator can be obtained as follows:

$$L = T - U = \frac{1}{2}(m_1L_1^2 + m_2L_2^2 + I_1 + I_2)\dot{\theta}^2 + I_2\dot{\theta}\dot{\phi} + \frac{1}{2}I_2\dot{\phi}^2 - (m_1L_1 + m_2L_2)g\cos\theta \tag{3}$$

Lagrange equations:

$$\begin{cases} \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial\dot{\theta}}\right) - \dfrac{\partial L}{\partial\theta} = 0 \\ \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial\dot{\phi}}\right) - \dfrac{\partial L}{\partial\phi} = T_r \end{cases} \Leftrightarrow \begin{cases} (m_1L_1^2 + m_2L_2^2 + I_1 + I_2)\ddot{\theta} \\ +I_2\ddot{\phi} - (m_1L_1 + m_2L_2)g\sin\theta = 0 \\ I_2(\ddot{\theta} + \ddot{\phi}) = T_r \end{cases} \tag{4}$$

When θ is very small ($\sin\theta \approx \theta$), we can obtain the linearized equation of the system as follows:

$$\begin{cases} (m_1L_1^2 + m_2L_2^2 + I_1 + I_2)\ddot{\theta} \\ +I_2\ddot{\phi} - (m_1L_1 + m_2L_2)\theta = 0 \\ I_2(\ddot{\theta} + \ddot{\phi}) = T_r \end{cases} \tag{5}$$

Let $x = [x_1 \quad x_2 \quad x_3 \quad x_4]^T = [\theta \quad \phi \quad \dot{\theta} \quad \dot{\phi}]^T$ be the state-variables vector. We obtain state-space equations of system as below:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ b/a & 0 & 0 & 0 \\ -b/a & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -1/a \\ (a+I_2)/(aI_2) \end{bmatrix} T_r \tag{6}$$

where $a = m_1L_1^2 + m_2L_2^2 + I_1$; $b = (m_1L_1 + m_2L_2)g$

Next, we have the model of DC motor with gear system:

$$\begin{cases} V = L_m \dfrac{di}{dt} + R_m i + K_e \omega_m \\ T_m = K_t i \\ T_r = N_g T_m \end{cases} \tag{7}$$

where: V is motor supply voltage, K_e is a motor back emf constant, ω_m is motor angular speed, L_m is armature coil inductance, R_m is armature coil resistance, i is armature current, T_m is the motor generated torque, K_t is a motor torque constant, and N_g is the gear ratio.

We can neglect the term of inductance because the motor inductance value is much less than the motor resistance value ($L_m << R_m$). The relationship between the motor and reaction wheel can be expressed as:

$$\omega_m = N_g \omega_r \tag{8}$$

where ω_r is reaction wheel angular speed.

Thus, the momentary torque that the motor supply for reaction wheel can be found as:

$$T_r = N_g K_t \left(\frac{V - K_e N_g \dot{\phi}}{R_m} \right) \tag{9}$$

By combining the mathematical equation of the motor and the reaction wheel, we can obtain the state form of the whole system:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ a_{31} & 0 & 0 & a_{34} \\ a_{41} & 0 & 0 & a_{44} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ b_3 \\ b_4 \end{bmatrix} V \; ; y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \tag{10}$$

where: $a_{21} = \frac{b}{a}$, $a_{24} = \frac{K_t K_e N_g^2}{aR_m}$, $a_{41} = -\frac{b}{a}$, $a_{44} = -\left(\frac{a+I_2}{aI_2}\right)\left(\frac{K_t K_e N_g^2}{R_m}\right)$, $b_3 = -\frac{K_t N_g}{aR_m}$, $b_4 = \left(\frac{a+I_2}{aI_2}\right)\frac{K_t N_g}{R_m}$

By substituting parameter values in Table 1, the state space expression of the system is changed into:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 19.2934 & 0 & 0 & 0.1457 \\ -19.2934 & 0 & 0 & -9.1500 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -0.2545 \\ 15.9825 \end{bmatrix} V \tag{11}$$

III. RESULTS AND DISCUSSIONS

3.1. LQR Controller

Suppose that the state equation of the system is:

$$\dot{x} = Ax + Bu \tag{12}$$

$$J = \frac{1}{2}\int_0^\infty (x^T Q x + u^T R u) dt \tag{13}$$

where Q is a positive semi definite matrix , R is a positive definite matrix

The optimal controller can make the system comeback to zero state and minimize the performance index *J*.

$$u(t) = -Kx(t) \tag{14}$$

where K is a state feedback gain matrix:

$$K = R^{-1}B^{T}S \tag{15}$$

And *S* is the solution of Riccati matrix algebraic equation:

$$A^{T}S + SA - SBR^{-1}B^{T}S + Q = 0 \tag{16}$$

In this paper, authors choose Q = [1 0 0 0; 0 1 0 0; 0 0 1 0; 0 0 0 1] and $R = 1$.

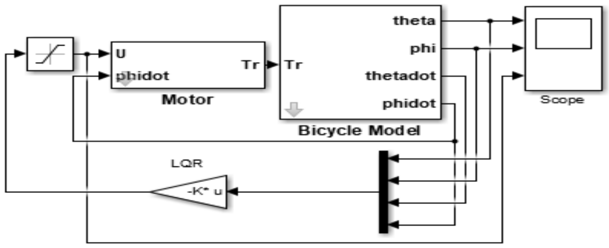


Figure 2. The scheme of LQR controller

3.2. Fuzzy-LQR Controller

3.2.1. Variables combination

The main idea is to use combining technique to make transform variables of the system into error (E) and the rate of change of error (EC).

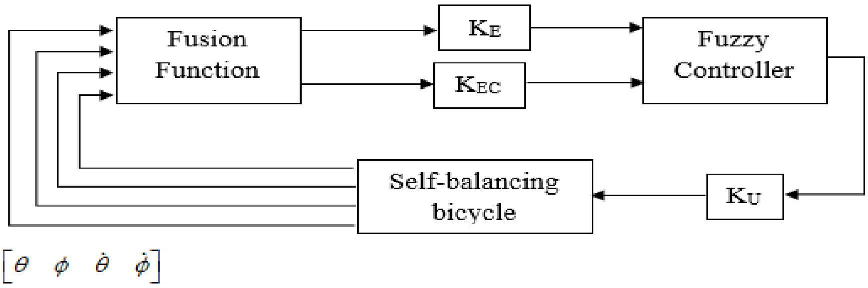


Figure 3. The model of fuzzy-LQR controller

Fig. 3 describes structure of Fuzzy-LQR controller, in which *KE*, *KEC*, *KU* are standardization coefficients.

Combination function is defined as:

$$F(X) = \begin{bmatrix} \frac{K_\theta}{\|K\|_2} & \frac{K_\phi}{\|K\|_2} & 0 & 0 \\ 0 & 0 & \frac{K_{\dot{\theta}}}{\|K\|_2} & \frac{K_{\dot{\phi}}}{\|K\|_2} \end{bmatrix} \tag{17}$$

where $K = \begin{bmatrix} K_\theta & K_\phi & K_{\dot{\theta}} & K_{\dot{\phi}} \end{bmatrix}$ is state feedback matrix collected by above LQR problem.

$$\|K\|_2 = \sqrt{K_\theta^2 + K_\phi^2 + K_{\dot{\theta}}^2 + K_{\dot{\phi}}^2} \tag{18}$$

Substitute K = [-935.9310 -1.0000 -214.5207 -2.2370] into (17):

$$F(X) = \begin{bmatrix} -0.9747 & -0.0010 & 0 & 0 \\ 0 & 0 & -0.2234 & -0.0023 \end{bmatrix}$$

The error E and the rate of change of error are calculated by this:

$$\begin{bmatrix} E \\ EC \end{bmatrix} = F(X)X^{T} \tag{19}$$

In this case:

$$\begin{cases} E = K_\theta \theta + K_\phi \phi \\ EC = K_{\dot{\theta}} \dot{\theta} + K_{\dot{\phi}} \dot{\phi} \end{cases} \tag{20}$$

Figure 1 shows the reaction scheme of the syntheses of the derivatives of ibuprofen through the esterification reaction, which resulted in the production of 5 derivatives.

3.2.2. Fuzzy-LQR controller

The fuzzy controller has 2 inputs E and EC, 1 output U. The input and output membership functions are triangular and have 7 functions each as shown in Figure 4, Figure 5 and Figure 6

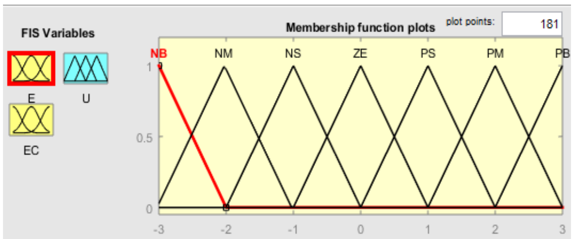


Figure 4. The membership functions of error E

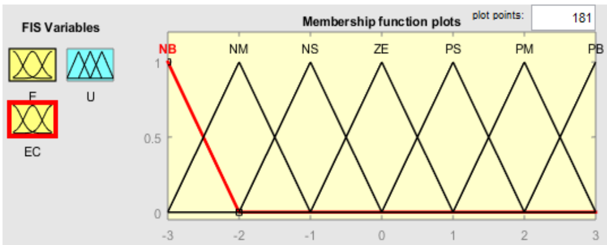


Figure 5. The membership functions of error EC

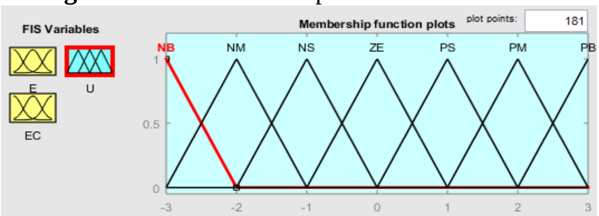


Figure 6. The membership functions of the power supply U

EC \ E	NB	NM	NS	ZE	PS	PM	PB
NB	NB	NB	NB	NM	NM	NS	ZE
NS	NB	NB	NM	NM	NS	ZE	PS
NM	NB	NM	NM	NS	ZE	PS	PM
ZE	NM	NM	NS	ZE	PS	PM	PM
PS	NM	NS	ZE	PS	PM	PM	PB
PM	NS	ZE	PS	PM	PM	PB	PB
PB	ZE	PS	PM	PM	PB	PB	PB

Table 2: Fuzzy set of rules of Fuzzy controller

With the built membership functions, we have the fuzzy set of rules in Table 2

The scheme of Fuzzy-LQR controller is shown in Figure 7

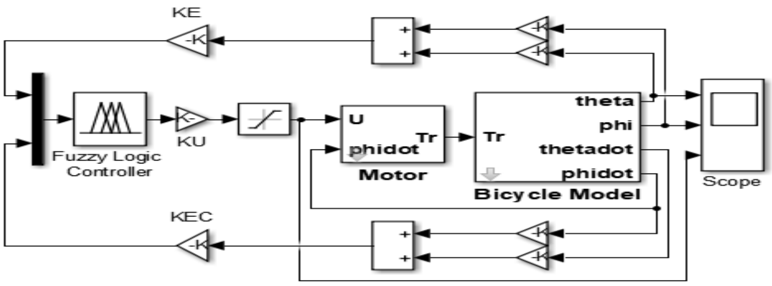
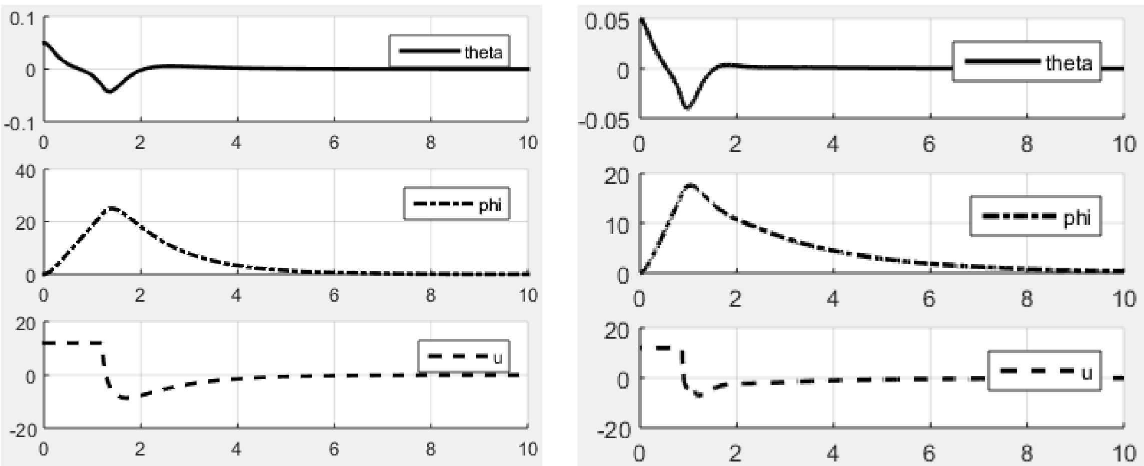


Figure 7. The scheme of Fuzzy-LQR controller

IV. SIMULATION RESULTS

The initial values of $\begin{bmatrix} \theta & \phi & \dot{\theta} & \dot{\phi} \end{bmatrix}$ are: $\begin{bmatrix} 0.05 & 0 & 0.005 & 0 \end{bmatrix}$



Figu

Figure 8. The simulation result of LQR controller

re 9. The simulation result of Fuzzy-LQR controller

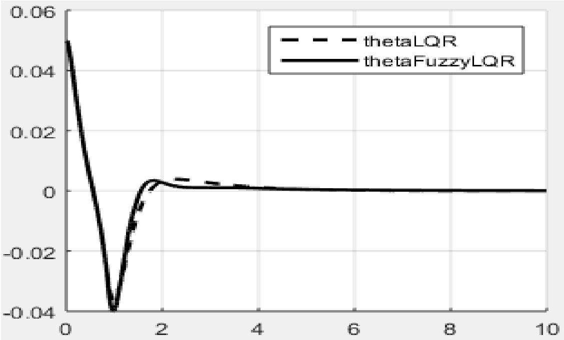


Figure 10. The comparison of simulation results of LQR and Fuzzy-LQR controller

Table 3: Performance criteria of 2 controllers in simulation

Criterion	LQR	Fuzzy-LQR
Absolute maximum overshoot (rad)	0.0432	0.0400
Settling time (s)	3.81	2.04

The results in Fig.8 and Fig.9 show that both LQR and Fuzzy-LQR controller can make the bicycle balance well. However, in comparison of the response of the angle between the bicycle and the vertical upright direction of 2 cases, Fig.10 and Table 3 show that Fuzzy-LQR controller gave a better result, with both settling time and absolute maximum overshot are smaller than which of LQR controller.

V. CONCLUSION

In this paper, we studied control of a self-balancing bicycle with a reaction wheel using LQR and Fuzzy-LQR algorithms. The simulation results showed that Fuzzy-LQR controller can control balance of bicycle well. Besides, performance index of Fuzzy-LQR controller is better than LQR controller. Therefore, we are building a real model of self-balancing bicycle in order to validate our control law that we've presented.

Conflict of interest

There is no conflict to disclose.

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