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# Non-vacuum solutions of five dimensional Bianchi type-I space-time in $f(R)$ theory of gravity

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***ABSTRACT:*** *The present paper is dealt with the study of the exact non-vacuum perfect fluid solutions of five dimensional Bianchi-type-I space-time in the metric version of $f(R)$ gravity considering stiff matter to obtain energy density and pressure of the universe on the lines of M. Sharif and M. Farasat Shamir (2010). In particular, we obtain two exact solutions which correspond to two models of the universe in higher five dimensional Bianchi type-I space-time. The function $f(R)$ are also evaluated for both the models. Finally the physical properties of these models have been discussed.*

***KEYWORDS:*** *Higher order theory of gravitation, $f(R)$ theory of gravity, Five dimensional Bianchi type-I Space-time.*

## I. INTRODUCTION

S. N. Pandey (2008) has developed a higher order theory of gravitation based on a Lagrangian density consisting of a polynomial of scalar curvature $R$ to obtain gravitational wave equations conformally flat. Recently S. N. Pandey and B. K. Sinha (2009) have studied spherically symmetric metric in the field equations of higher order theory of gravitation which is obtained by modifying Einstein's field equations in general relativity theory. Thus Einstein's general relativity is nothing but the particular case of this higher order modified theory i.e, $f(R)$ theory of gravity.

Many authors have obtained spherical, cylindrical and plane symmetric solutions in $f(R)$ gravity [Hollenstein and Lobo (2008), Azadi et al.(2008), M. Sharif and M. Farasat Shamir (2009), etc.]. Sotiriou and Faraoni (2010) presented some important aspects of $f(R)$ theory of gravity in Metric, Palatini and Metric-Affine formalisms.

Recently, in the paper [1], M. Sharif and M. Farasat Shamir (2010) have studied the non-vacuum perfect fluid solutions of four dimensional Bianchi type-I space-time in the framework of metric $f(R)$ gravity. For this purpose they have considered stiff matter to find energy density and pressure of the universe and found two exact solutions which correspond to two models of the universe and observed that the first solution yields a singular model while the second solution gives a non-singular model. Finally the physical behavior of these models has been discussed using some physical quantities. The function $f(R)$ of the Ricci scalar is evaluated also by them. It is noted that the work of M. Sharif and M. Farasat Shamir (2010) is in the framework of four dimensional Bianchi type-I space-time. In the present paper we wish to investigate the similar situation in higher five dimensional Bianchi type-I space-time.

In a past few years there have been many attempts to construct a unified field theory based on the idea of multidimensional space-time. The idea that space-time should be extended from four to higher five dimension was introduced by Kaluza and Klein (1921, 26) to unify gravity and electromagnetism. Several aspects of five dimensional space-time have been studied in different theories by many authors [Wesson (1983, 84), Reddy D.R.K. (1999),
Khadekar et al. (2001), Ghosh and Dadhich (2001), Adhao (1994), Thengane (2000), Ambatkar (20002), Jumale(2006) etc.].

We observed that the four dimensional work of M. Sharif and M. Farasat Shamir (2010) regarding non-vacuum perfect fluid solutions in the metric version of $f(R)$ gravity can further be extended to the higher five dimensional Bianchi type-I space-time and therefore, an attempt has been made in the present paper.

The paper is organized as follows : In section-2, we briefly give the five dimensional field equations in metric $f(R)$ gravity. Section-3 is used to find exact non-vacuum solutions of Bianchi type-I space-time in $V_5$, section-4 is dealt with some physical quantities. Section-5 and 6 are dealt with five dimensional models of the universe and in the last section-7, we summarize and conclude the results.

## II. FIVE DIMENSIONAL FIELD EQUATIONS IN $f(R)$ THEORY OF GRAVITY

The five dimensional field equations in $f(R)$ theory of gravity are given by :

$$F(R)R_{ij} - \frac{1}{2} f(R) g_{ij} - \nabla_i \nabla_j F(R) + g_{ij} \Box F(R) = kT_{ij}, \quad (i, j = 1,2,3,4,5) \tag{1}$$

where $F(R) \equiv \frac{df(R)}{dR}$, $\Box \equiv \nabla^i \nabla_i$, $\nabla_i$ is the covariant derivative and $T_{ij}$ is the standard matter energy momentum tensor.

Contracting the above field equations we have

$$F(R)R - \frac{5}{2} f(R) + 4 \Box F(R) = kT . \tag{2}$$

Using this equation in (1), the field equations take the form

$$F(R)R_{\mu\nu} - \nabla_\mu \nabla_\nu F(R) - kT_{\mu\nu} = \frac{1}{5}[F(R)R \Box F(R) - kT] g_{\mu\nu} . \tag{3}$$

In this way we have eliminated $f(R)$ from the field equations and therefore, the equation (3) helps us to solve the field equations.

## III. EXACT NON-VACUUM SOLUTIONS OF BIANCHI TYPE - I SPACE-TIME IN $V_5$

In this section we find exact non vacuum solutions of five dimensional Bianchi type-I space time in $f(R)$ theory of gravity. The line element of Bianchi type-I space-time in $V_5$ is given by

$$ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)(dy^2) - C^2(t)dz^2 + D^2(t)du^2 \tag{4}$$

where $A, B$ and $C$ are cosmic scale factors. The corresponding Ricci scalar is

$$R = -2\left[\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\ddot{D}}{D} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{A}\dot{D}}{AD} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{B}\dot{D}}{BD} + \frac{\dot{C}\dot{D}}{CD}\right], \tag{5}$$

where dot means derivative with respect to $t$. The energy momentum tensor for perfect fluid gives

$$T_{ij} = (\rho + p)u_i u_j - pg_{ij} \tag{6}$$

satisfying the equation of state

$$p = w\rho, \qquad 0 \le w \le 1, \tag{7}$$

where $\rho$ and $p$ are energy density and pressure of the fluid while $u_i = \sqrt{g_{00}}(1,0,0,0,0)$ is the five-velocity in co-moving co-ordinates. Since the metric (4) depends only on $t$, equation (3) yields a set of differential equation for $F(t)$, $A, B, C, \rho$ and $p$. Thus the subtraction of the 00-component and 11-component gives

$$-\frac{\ddot{B}}{B}-\frac{\ddot{C}}{C}-\frac{\ddot{D}}{D}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{A}\dot{D}}{AD}+\frac{\dot{A}\dot{F}}{AF}-\frac{\ddot{F}}{F}-\frac{k}{F}(\rho+p)=0\,. \tag{8}$$

Similarly the subtraction of the 00-component and 22- component gives

$$-\frac{\ddot{A}}{A}-\frac{\ddot{C}}{C}-\frac{\ddot{D}}{D}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{B}\dot{D}}{BD}+\frac{\dot{B}\dot{F}}{BF}-\frac{\ddot{F}}{F}-\frac{k}{F}(\rho+p)=0\,. \tag{9}$$

The subtraction of the 00-component and 33- component gives

$$-\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B}-\frac{\ddot{D}}{D}+\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{C}\dot{D}}{CD}+\frac{\dot{C}\dot{F}}{CF}-\frac{\ddot{F}}{F}-\frac{k}{F}(\rho+p)=0\,, \tag{10}$$

and the subtraction of the 00-component and 44- component gives

$$-\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B}-\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{D}}{AD}+\frac{\dot{B}\dot{D}}{BD}+\frac{\dot{C}\dot{D}}{CD}+\frac{\dot{D}\dot{F}}{DF}-\frac{\ddot{F}}{F}-\frac{k}{F}(\rho+p)=0\,. \tag{11}$$

It is interesting to note that there are three independent differential equations in five dimensional case also.

The conservation equation, $T^{ij}_{;j}=0$ leads to

$$\dot{\rho}+(p+\rho)\left[\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}+\frac{\dot{D}}{D}\right]=0\,. \tag{12}$$

Thus we have four differential equations with six unknowns namely $A$, $B$, $C$, $F$, $\rho$ & $p$.

From equations (8) - (9), (9) -(10) and (8) - (10), we get respectively

$$-\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B}+\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)+\left(\frac{\dot{C}}{C}+\frac{\dot{D}}{D}+\frac{\dot{F}}{F}\right)=0\,, \tag{13}$$

$$-\frac{\ddot{B}}{B}-\frac{\ddot{C}}{C}+\left(\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right)\left(\frac{\dot{A}}{A}+\frac{\dot{D}}{D}+\frac{\dot{F}}{F}\right)=0\,, \tag{14}$$

$$\frac{\ddot{C}}{C}-\frac{\ddot{D}}{D}+\left(\frac{\dot{C}}{C}-\frac{\dot{D}}{D}\right)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{F}}{F}\right)=0\,. \tag{15}$$

$$-\frac{\ddot{D}}{D}-\frac{\ddot{A}}{A}+\left(\frac{\dot{D}}{D}-\frac{\dot{A}}{A}\right)\left(\frac{\dot{B}}{B}+\frac{\dot{C}}{C}+\frac{\dot{F}}{F}\right)=0\,. \tag{16}$$

These equations imply that

$$\frac{B}{A} = d_1 \exp[\ c_1 \int \frac{dt}{a^4 F}] , \tag{17}$$

$$\frac{C}{B} = d_2 \exp[\ c_2 \int \frac{dt}{a^4 F}] , \tag{18}$$

$$\frac{D}{C} = d_3 \exp[\ c_3 \int \frac{dt}{a^4 F}] \tag{19}$$

$$\frac{A}{D} = d_4 \exp[\ c_4 \int \frac{dt}{a^4 F}] \tag{20}$$

where $c_1, c_2, c_3, c_4$ and $d_1, d_2, d_3, d_4$ are constants of integration which satisfy the relation

$$c_1 + c_2 + c_3 + c_4 = 0 , \quad d_1 d_2 d_3 d_4 = 1 . \tag{21}$$

Using equation (17), (18), (19) and (20), we can write the metric functions explicitly as

$$A = ap_1 \exp[\ q_1 \int \frac{dt}{a^4 F}] , \tag{22}$$

$$B = ap_2 \exp[\ q_2 \int \frac{dt}{a^4 F}] , \tag{23}$$

$$C = ap_3 \exp[\ q_3 \int \frac{dt}{a^4 F}] , \tag{24}$$

$$D = ap_4 \exp[\ q_4 \int \frac{dt}{a^4 F}] , \tag{25}$$

where $p_1 = (d_1^{-3} d_2^{-2} d_3^{-1})^{1/4}$, $\quad p_2 = (d_1 d_2^{-2} d_3^{-1})^{1/4}$, $\quad p_3 = (d_1 d_2^{2} d_3^{-1})^{1/4}$, $\quad p_4 = (d_1 d_2^{2} d_3^{3})^{1/4}$ (26)

and

$$q_1 = -\frac{3c_1 - 2c_2 - c_3}{4} , \quad q_2 = \frac{c_1 - 2c_2 - c_3}{4} , \quad q_3 = \frac{c_1 + 2c_2 - c_3}{4} , \quad q_4 = \frac{c_1 + 2c_2 + 3c_3}{4} \tag{27}$$

$c_i$ and $d_i$ are constants of integration. Using power law relation between $F$ and $a$ we have

$$F = ka^m , \tag{28}$$

where $k$ is the constant of proportionality, $m$ is any integer ( here taken as -3 ) and $a$ is given by

$$a = (nlt + k_1)^{1/n} , \qquad n \neq 0$$

$$a = k_2 \exp(\ lt ) , \qquad n = 0 , \tag{29}$$

where $k_1$ and $k_2$ are constants of integration. It is mentioned here that we have used $H = la^{-n}$, $l > 0$, $n \geq 0$ to get the above equation. Thus we obtain two values of the average scale factor corresponding to two different models of the universe.

## IV. SOME IMPORTANT PHYSICAL QUANTITIES

In this section we define some important physical quantities

The average scale factor and the volume scale factors are defined respectively as under :

$$a = (ABCD)^{\frac{1}{4}}, \quad V = a^4 = ABCD \quad . \tag{30}$$

The generalized mean Hubble parameter $H$ is defined by

$$H = \frac{1}{4}[H_1 + H_2 + H_3 + H_4], \tag{31}$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$, $H_3 = \frac{\dot{C}}{C}$, $H_4 = \frac{\dot{D}}{D}$ are the directional Hubble parameters in the directions of $x, y, z$ and $u$ axis respectively. Using equations (30) and (31), we obtain

$$H = \frac{1}{4}\frac{\dot{V}}{V} = \frac{1}{4}[H_1 + H_2 + H_3 + H_4] = \frac{\dot{a}}{a}. \tag{32}$$

The mean anisotropy parameter $\overline{A}$ is given by

$$\overline{A} = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{H_i - H}{H}\right)^2 . \tag{33}$$

The expansion scalar $\theta$ and shear scalar $\sigma^2$ are defined as under

$$\theta = u^i_{;i} = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{2\dot{C}}{C}, \tag{34}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} \tag{35}$$

where $\sigma_{ij} = \frac{1}{2}(\nabla_j u_i + \nabla_i u_j) - \frac{1}{4}g_{ij}\theta$ . (36)

In thermodynamics, the entropy of the universe is given by

$$Tds = d(\rho V) + pdV \quad . \tag{37}$$

## V. FIVE DIMENSIONAL MODEL OF THE UNIVERSE WHEN $n \neq 0$

For this model, $F$ becomes $F = k(nlt + k_1)^{-3/n}$ and the corresponding metric coefficients $A, B$ and $C$ turn out to be

$$A = p_1(nlt + k_1)^{1/n} \exp[\frac{q_1(nlt + k_1)^{\frac{n-1}{n}}}{kl(n-1)}], \qquad n \neq 1 \tag{38}$$

$$B = p_2(nlt + k_1)^{1/n} \exp[\frac{q_2(nlt + k_1)^{\frac{n-1}{n}}}{kl(n-1)}], \qquad n \neq 1 \tag{39}$$

$$C = p_3(nlt + k_1)^{1/n} \exp[\frac{q_3(nlt + k_1)^{\frac{n-1}{n}}}{kl(n-1)}], \qquad n \neq 1 \tag{40}$$

$$D = p_4(nlt + k_1)^{1/n} \exp[\frac{q_4(nlt + k_1)^{\frac{n-1}{n}}}{kl(n-1)}], \qquad n \neq 1 \tag{41}$$

The mean generalized Hubble parameter and the volume scale factor become

$$H = \frac{l}{nlt + k_1}, \quad V = (nlt + k_1)^{4/n}. \tag{42}$$

The mean anisotropy parameter $\overline{A}$ turns out to be

$$\overline{A} = \frac{q_1^2 + q_2^2 + q_3^2 + q_4^2}{2k^2(nlt + k_1)^{2/n}}. \tag{43}$$

The expansion $\theta$ and shear scalar $\sigma^2$ are given by

$$\theta = \frac{4l}{nlt + k_1}, \qquad \sigma^2 = \frac{q_1^2 + q_2^2 + q_3^2 + q_4^2}{2k^2(nlt + k_1)^{2/n}}. \tag{44}$$

For stiff matter $(\omega = 1)$, were have $p = \rho$. Thus the energy density and pressure of the universe become

$$2kp = 2k\rho = \frac{-12kl^2}{(nlt + k_1)^{3/n+2}} - \frac{q_1^2 + q_2^2 + q_3^2 + q_4^2}{k(nlt + k_1)^{5/n}}. \tag{45}$$

The entropy of universe is given by

$$Tds = \frac{1}{k}[6kl^3(2n-5)(nlt + k_1)^{1/n-3} - \frac{3l}{2k}(q_1^2 + q_2^2 + q_3^2 + q_4^2)(nlt + k_1)^{-1/n-1}. \tag{46}$$

Also equation (12) leads to

$$\rho = \frac{c}{V^2}, \tag{47}$$

where $c$ is an integration constant. It is mentioned here that this value of $\rho$, when compared with the value obtained in equation (41), gives a constraint

$$kc + 6kl^2 = 0 \tag{48}$$

Which holds only when $n = 5/2$ and $q_1^2 + q_2^2 + q_3^2 + q_4^2 = 0$ . The function of Ricci scalar, $f(R)$ is

$$f(R) = \frac{2k}{5}(nlt + k_1)^{-3/n} R + \frac{12}{5} kl^2 (2n - 5)(nlt + k_1)^{-2n-3/n}, \tag{49}$$

where $R \equiv R_1 = 4l^2(2n-5)(nlt + k_1)^{-2}$ . For a special case $n = \frac{1}{2}$, $f(R)$ turns out to be

$$f(R) = \frac{2k}{5}(-\frac{R}{16l^2})^{3/2n} - \frac{48}{5}kl^2(-\frac{R}{16l^2})^{3/2n+1} \tag{50}$$

which gives $f(R)$ in terms of $R$ .

## VI. FIVE DIMENSIONAL MODEL OF THE UNIVERSE WHEN $n = 0$

Here the metric coefficient take the form

$$A = p_1 k_2 \exp(lt) \exp[-\frac{q_1 \exp(-lt)}{klk_2}], \tag{51}$$

$$B = p_2 k_2 \exp(lt) \exp[-\frac{q_2 \exp(-lt)}{klk_2}], \tag{52}$$

$$C = p_3 k_2 \exp(lt) \exp[-\frac{q_3 \exp(-lt)}{klk_2}]. \tag{53}$$

$$D = p_4 k_2 \exp(lt) \exp[-\frac{q_4 \exp(-lt)}{klk_2}]. \tag{54}$$

The mean generalized Hubble parameter will become

$$H = l \tag{55}$$

while the volume scale factor turns out to be

$$V = k_2^4 \exp(4lt). \tag{56}$$

The mean anisotropy parameter $\bar{A}$ becomes

$$\bar{A} = [\frac{q_1^2 + q_2^2 + q_3^2 + q_4^2}{4l^2 k^2 k_2^2}] \exp(-2lt) \tag{57}$$

while the quantizes $\theta$ and $\sigma^2$ are given by

$$\theta = 4l, \qquad \sigma^2 = [\frac{q_1^2 + q_2^2 + q_3^2 + q_4^2}{2k^2 k_2^2}] \exp(-2lt). \tag{58}$$

For stiff matter, the energy density and pressure turn out to be

$$2k\rho = 2kp == \frac{-12\,kl^2 \exp(-3lt)}{k_2^3} - [\frac{q_1^2 + q_2^2 + q_3^2 + q_4^2}{kk_2^5}]\exp(-5lt)\ . \tag{59}$$

The corresponding entropy is

$$Tds = \frac{1}{k}[-30\,kk_2 l^3 \exp(lt) - \frac{3l}{2kk_2}(q_1^2 + q_2^2 + q_3^2 + q_4^2)\exp(-lt)]\ . \tag{60}$$

The constraint equation with the condition, $q_1^2 + q_2^2 + q_3^2 + q_4^2 = 0$ , is given by

$$kc - 12\,kk_2^5 l^2 \exp(5lt) = 0\ . \tag{61}$$

The function of Ricci scalar, $f(R)$ , takes the form

$$f(R) = \frac{2k}{5k_2^3}\exp(-3lt)(R - 30\,l^2)\ . \tag{62}$$

which reduces to $f(R) = \frac{2}{5}[\frac{12\,l^2 k^{8/3}}{kC}]^{3/5}(R - 30\,l^2)$ (63)

using the constraint equation (62). This corresponds to the general function $f(R)$

$$f(R) = \sum a_n R^n\ , \tag{64}$$

where $n$ may take values from negative or positive.

## VII. CONCLUDING REMARK

(i) In this paper we have studied the expansion of the universe in metric $f(R)$ theory of gravity. Using the non-vacuum field equations, we have obtained exact solutions of the Bianchi type-I space-time. These exact solutions correspond to two models of the universe. For $n \neq 0$ , we have obtained singular model of the universe and for $n = 0$ we found a non-singular model. For these solutions we have evaluated some important cosmological physical quantities such as expansion scalar $\theta$ , shear scalar $\sigma^2$ and mean anisotropy parameter $\overline{A}$ . The entropy of the universe is also found.

Our observations regarding the model of the universe are as under

**(i) For $n \neq 0$ , singular model of the universe**

For this model, we have a singularity at $t = t_s = -\frac{k_1}{nl}$ .

The physical parameters $H_1, H_2, H_3, H_4\ H$ , $\theta$ and $\sigma^2$ are all infinite at this point for $n > 0$ but volume scale factor vanishes.

The mean anisotropic parameter $\overline{A}$ is also infinite at this point for $0 < n < 1$ and it will vanishes $n > 1$ . The function of the Ricci scalar $f(R)$ , energy density $\rho$ , pressure $p$ and $T$ are also infinite while the metric function $A, B$ and $C$ vanish at this point of singularity.

The model suggests that expansion scalar $\theta$ and shear scalar $\sigma^2$ decrease for $n > 0$ with the time. The mean anisotropic parameter also decreases for $n > 1$ with the increase in time.
This indicate that after a large time the expansion will stop completely and universe will achieve isotropy.

The isotropy condition i.e, $\sigma^2 / \theta \to 0$ as $t \to \infty$, is also satisfied. The entropy of the universe is infinite for $n > 1/3$ .

*Thus we can conclude from these observations that the model starts its expansion from zero volume with infinite energy density and pressure at* $t = t_s$ *and it continues to expand with time.*

**(ii) For $n = 0$, non-singular model of the universe**

The physical parameters $H_1, H_2, H_3, H_4, H$, $\sigma^2$ and $A$ are all finite for all finite values of $t$.

The mean generalized Hubble parameter $H$ and expansion scalar $\theta$ is constant while $f(R)$ is also finite here.

The metric functions $A, B$ and $C$ do not vanish for this model.

The entropy of the universe is finite.

The energy density and pressure become infinite as $t \to -\infty$ which shows that the universe started its evolution in an infinite past with a strong pressure and energy density.

The isotropy condition is also verified for this model.

*The volume scale factor increases exponentially with time which indicates that the universe starts its expansion with zero volume from infinite past.*

We observed that all the four dimensional results of Sharif and Farasat (2010) can be obtained from our investigations after the reduction of dimension.. Therefore it is pointed out that the solution of Sharif and Farasat (2010) is a particular case of solution presented here.

We think that this new exact higher dimensional solution should bring some additional information and therefore, they need to be further investigated.

## VIII. ACKNOWLEDGEMENT

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