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Analytical Approach and Mechanical Estimate of Vibration Behavior in Presence Of Ball Bearings

EL HAD Khalid[1,2], HACHIM Abdelilah[1,2], EL AMRI Abdelhamid[1], ABOUELAZ Ahmed[3], EL GHORBA Mohamed[1],

[1] *Laboratory Mechanical Control and Characterization of Materials and Structures,, BP 8118, ENSEM, Casablanca, Morocco*
[2]*Higher Institute of Maritime Studies, Department of Machinery, 7 km Road El Jadida, Hassan II University, BP 8118 Oasis Casablanca, Morocco*
[3]*University Hassan II, Faculty of Sciences Ain Chock, laboratory of mechanics, BP 5366 Maarif, Casablanca, Morocco.*

ABSTRACT: *Whether for industrial, transportation or complex systems, ball bearing are considered as major factor in kinematics rotating systems. Apart from their strategic role in the drive assemblies, these mechanical components can require an increased rate of reliability.Bearings turbines and rotating machines are primordial in most mechanical components which contribute directly to the performance of automotive and aerospace engines through reliability. Due to their role as liaison between fixed and moving parts, any failure could have catastrophic consequences such as loss of engine use.This research is aimed to develop the analytical fundamentals details of new model which is dedicated to the dynamic behavior of rotating ball bearings. Theoretical research will be detailed enough to provide a simple model that lends easily to programming, but powerful enough to incorporate the effect of a maximum of influential parameters on the vibration of bearings.The essence of this work is to present an analytical model of a specific type single of row effect balls (SKF 6004). This study is a first step dedicated to define the geometric characteristics, and in second time to determine the equations governing the distribution of responsibilities within the ball bearing.*

KEYWORDS: *ball bearing, rotating systems, reliability, dynamic behavior, vibration.*

I. INTRODUCTION

Even if the geometry of a ball bearing is perfect, it will still produce vibrations. The vibrations are caused by the rotation of a finite number of loaded rolling contacts between the balls and the guiding rings. Because these contacts are elastic, the bearing stiffness becomes explicitly dependent on time.In general, a time varying stiffness causes vibrations, even in the absence of external loads. Since the stiffness can be regarded as a system parameter, the variable stiffness leads to a so-called parametric excitation. It is one of the major sources of vibration in ball bearings. The first systematic research on this subject was conducted by (Perret, 1950) and (Meldau, 1951).

(Dowson and Higginson, 1977) have proposed the first analytical solutions to EHD in case of cylinder/plane contact. Then, in the middle of 1970s, digital solutions contact ellipsoid/ maps have been proposed by (Hamrock and Dowson, 1977). Thus, using these procedures, we can go back to the thicknesses of lubricant in contact and we can realize that these thicknesses vary between a few nanometers.From the viewpoint of contact fatigue in the presence of indentation, (Wang and al., 2007) exhibit a work based on the introduction of a ceramic rolling element (Si_3N_4) from the rolling ones in steel. This study demonstrates a “smoothing effect” created by the passage of ceramic rolling element which generates strong plastic deformation of bead. This “smoothing effect” makes it possible to reduce the height of beads and, at the same time, local overpressure. This suggests that the addition of ceramic allows increasing number of cycles to chipping adjacent indents.In addition, the work presented by (Jacq, 2001) shows a smaller effect of the load on damage by chipping in vicinity of the with respect to sliding.(Cheng, 1997) present a model including contact fatigue crack initiation in slip bands formed in the grains of the material. The slip bands are modeled by two stacks of dislocations of opposite signs, of which the length is equal to that of the grains. Accumulation of dislocations occurs during the cyclic loads due to irreversibility of dislocations movement. This type of analytical study is actually based on the work of (Tanaka and Mura, 1981), together with its collaborators studied crack initiation in a crystalline material subjected to cyclic loading.Moreover, the modeling of the propagation of cracks in contact fatigue was studied by many other authors. Of particular we note: (Keer and al., 1983), (Lee and al., 1986), (Murakami and al., 1994).

II. ROLLER BEARING

The bearing is a mechanical member permitting rotation of one part relative to another by reducing friction. It has three main functions:

- -The transmission of forces
- -Rotation states in one piece with minimum friction
- -The position of a shaft relative to the housing
- Thus, if we describe a ball, we find rolling elements, separated by a cage, an inner ring and an outer ring provided with raceways (Figure 1).

The ball bearing can replace the sliding friction by rolling one. However, various body contacting (inner ring and balls for example) showing differences speed in their contact areas that will generate local slipping. Indeed, the friction generated by the fastest surface will tend to pull the slow surface.

Figure 1: Overview of ball bearings components

To ensure a long life, the roll bearings are lubricated with grease or oil. According to their operating conditions (load, speed.......), different regimes of lubrication can be occurs:
- A system of hydrodynamic lubrication (HD)
- A system of elastohydrodynamic lubrication (EHD)

2.1. The E.H.D Contacts In Bearing

A roll bearing can force the position of shaft relative to system. The rolling elements transmit the forces applied by the shaft to the inner race fixed to the outer ring. They facilitate also the rotation of shaft relative to housing (Figure 2).

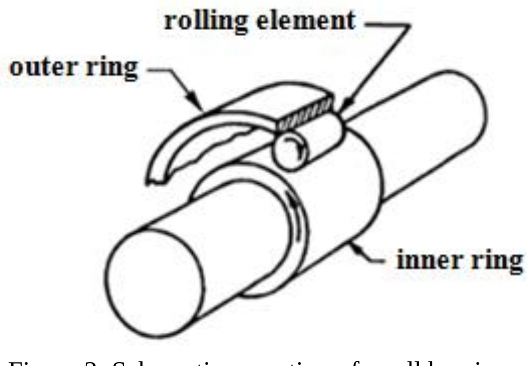

Figure 2: Schematic operation of a roll bearing

Geometry of rolling elements and raceways (inner and outer ring) differs depending on the loading conditions (axial and / or radial moments) and speed. Bearings angular contacts are designed to simultaneously collect radial and axial loads (Figure 3). They can operate at very high speed and are therefore used to stopping gas turbines. The axial component of the thrust and the radial component generated by gas is due to the own weight of the shaft and the unbalance effects engagement in the case of the use of a speed reducer.

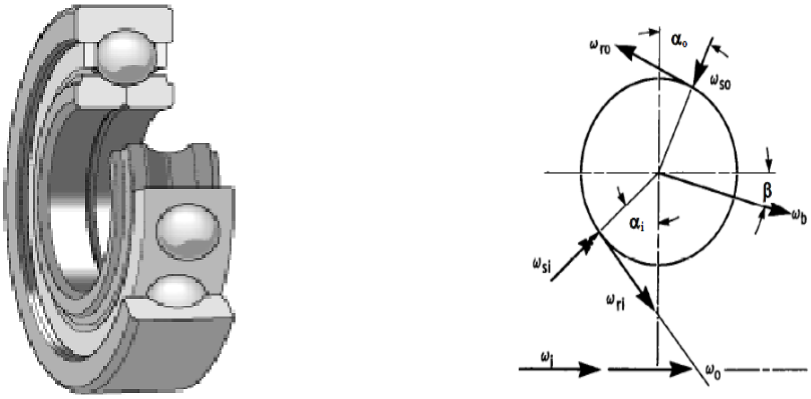

Figure 3: Ball bearing angular contact

Because of the high speeds (above 20000 trs/min) , centrifugal forces (Fc) acting on the balls have the same order of magnitude, as the forces associated with contact with the rings (Figure 4). These efforts are reflected in resolution of load distribution in the bearing, as the gyroscopic moments acting on the ball. The details of the corresponding quasi- static analysis are described by (Harris, 1984).The determination of bearing kinematics is based on assumption of pure rolling, which is defined by the equality of geometric points of speed ball and ring contact considering non-deformable solids. The spin rate of the balls (ω_b) and the orbital velocity (ω_m) depend on angles of contact between ball and the outer ring (α_o) and ball with inner ring (α_i) (Figure 4). Centrifugal forces tend to flatten ball to the bottom of outer ring and corresponding contact angle (α_o) is less than the angle of geometric contact (α_0). The opposite occurs for the inner contact angle (α_i). The pivot bearing is impossible without simultaneously ball contacts / inner ring and ball / outer ring.Due to centrifugal forces, pressures and contact surfaces are more important to touch ball / ball at the outer ring / inner ring contact. The pivot is then assumed to be zero, thus fixing the kinematics of the bearing. This is assuming control of the outer ring ball (Dawson and al. 1986).

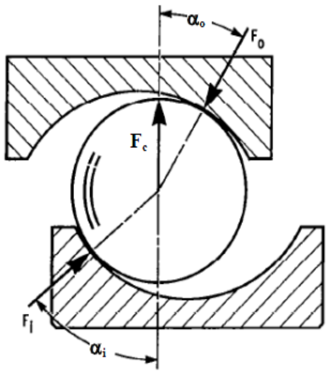

Figure 4: Distribution of kinematic and load bearing ball angular contact with high speed

The contact between the rolling elements and rings is characterized by non-compliant type ball / plane, where the local pressure has the effect of deforming the solid surfaces. Hertz has been the pioneer in this field (1881), laying the foundations of the Hertzian contact theory which estimates the distribution of pressure, deformation and dimension of contact.The speed differences between the solids in contact, due to sliding of pivot, are responsible of frictional forces, directly proportional to the coefficient of friction. This reflects the relationship between normal and transmitted tangential at contact forces.In the case of a dry contact, the friction coefficient is higher than coefficient of lubricated contact. It leads to friction levels which is too important in roll bearing and cause more or less long term ruin in operation.

2.2. The H.D. Contacts In The Bearing

In a ball bearing exposed to axial and radial forces, the charge distribution is not in cyclic symmetry. The contact angles are different for each of rolling elements which have not all the same kinematics.However, it is necessary that all rolling elements retain on average a regular spacing from each other to ensure the proper functioning of mechanism service. The cage can meet this need by positioning each element in a cell. It supports the efforts made by opponents of the printed contacts with rings (Figure 5) movement. The lubricant can accommodate the difference in speed between the rolling element and the cage and avoid a dry friction between the contacting surfaces.

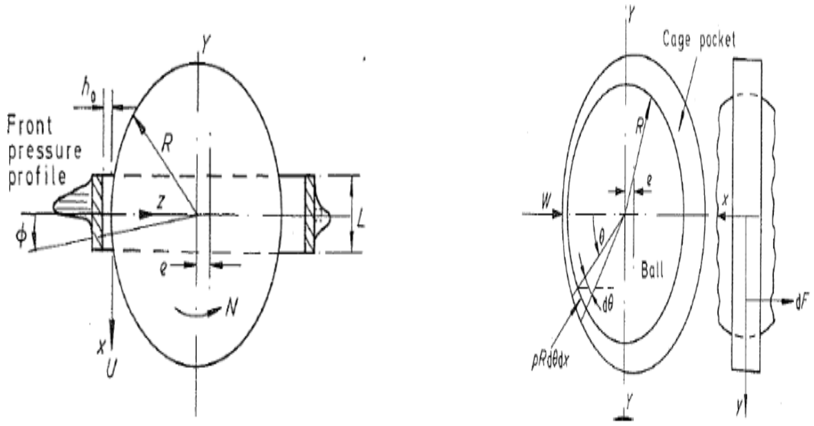

Figure 5: Description of contact ball cage in a ball bearing

The cage plays the role of separator and must, however, be guided in rotation especially in high speed applications where the imbalance caused by the eccentricity of gravity center can be fatal. It is possible to guide the cage by the inner ring or the outer ring (Figure 6). In the first case, the cage will tend to be driven (mobile ring) but is sensitive to unbalance and therefore requires a good balance. In the second case the cage is restrained by the outer ring (fixed) and is more sensitive to the phenomenon of torque (lash adjuster operation by differential centrifugation and expansion).

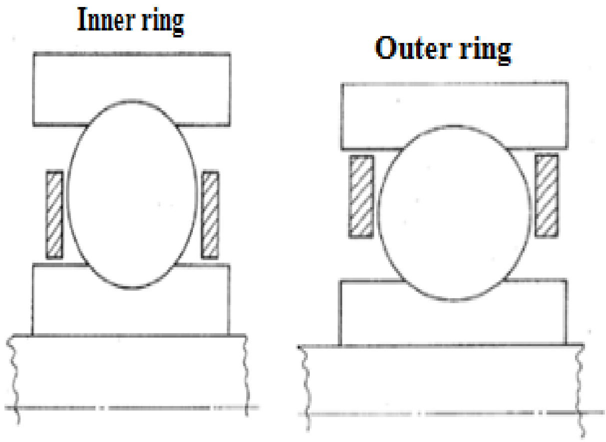

Figure 6 - Guiding of the cage and inner ring/outer ring

III. MATHEMATICAL DEVELOPMENT OF A SINGLE BEARING

3.1. INITIAL MODEL

In this part we consider a rotor, bearing rings and support. The model becomes complex with a large number of degrees of freedom (ddl). In our model, we applied three degree of freedom system as described in Figure 8, in order to focus the study on the vibration response of the balls (Sanchette-Gosset, 1993) and (Tanaka and Mura, 1981).The dynamic behavior of rigid bearing support and the contribution of the support is also considered. Thus, this simplified model that will be adjusted with the calculation of vibration response.

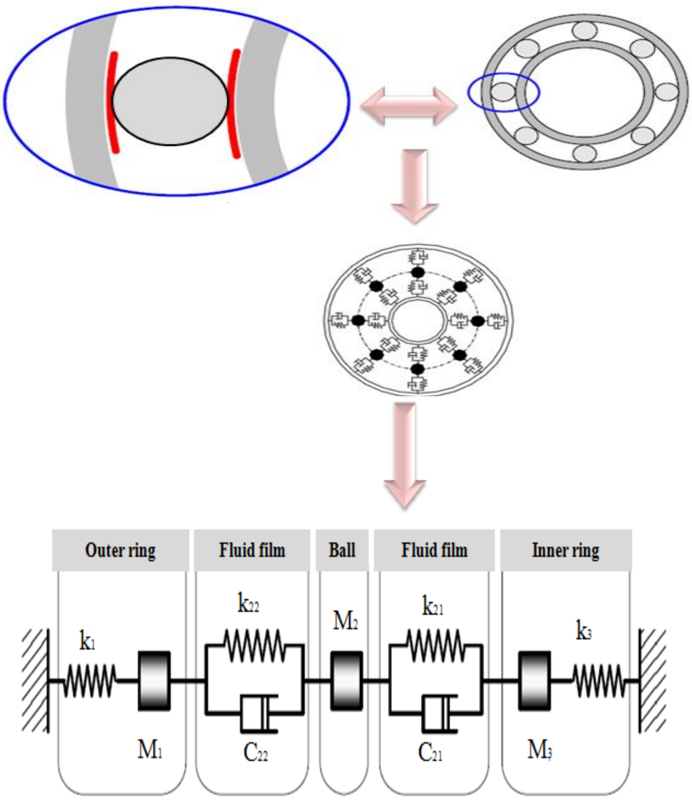

Figure 8: Model of bearing system

With:
M1: Mass of outer ring [kg]
M3: Mass of inner ring [Kg]
M2: Mass of ball [kg]
K1: stiffness of outer ring [N/m],
K3: stiffness of inner ring [N / m]
K21: Stiff ball [N / m]
C22: Damping ratio Film external fluid [N.s / m]
C21: Damping coefficient of internal fluid film [N.s / m]
The different parameters governing the behavior of this model were developed using the following analytical elements:

The Rings : We considered a mass-spring system with neglected structural damping, the rigidities of the two rings is determined by the following equations:

Rigidity of the inner ring $k_3 = M_3\omega_3^2$ (1)

Rigidity of the outer ring $k_1 = M_1\omega_1^2$ (2)

Each of inner and outer rings will be represented by a mass-spring combination which corresponds to a single degree of freedom system. The internal damping has been neglected. The natural frequency of the flexural vibration mode number "n" is given by the following expression:

$$\omega_n = \frac{n[n^2-1]}{\sqrt{1+n^2}}\sqrt{\frac{EI}{\mu R^4}} \quad (3)$$

where:
ωn angular frequency of mode [rad/s]
n   order mode
E   modulus longitudinal elasticity [N/m2]
I   moment of inertia cross section of the ring [m4]
μ   linear density [kg/m]

R radius of ring [m].

Modes 0 and 1 are considered as rigid modes. Therefore the second mode is considered as the first bending vibration mode. The angular frequency of the system to a single degree of freedom will be taken as:

$$\omega_n = 2.68\sqrt{\frac{EI}{\mu R^4}}$$

THE BALLS : Rigidity ball has the order of 8.3 109 N / m. Compared to other rigidities, it is considered infinitely rigid. Therefore modeling the ball was deemed to be a mass element.

THE FLUID FILM : The development of elastohydrodynamic theory of lubrication (EHD) showed that films with a few micrometers thick are occurs in rolling contact. Therefore, the characterization of lubrication system depends on the value of lubricant film thickness. The theory developed by Hamrock [1], expressed in terms of stiffness and damping coefficients dimension film of fluid is given by the following expressions:

Fluid film stiffness:

$$k = \frac{4}{W_r\lambda_k^2}\left[\frac{\varepsilon_0}{(1-\varepsilon_0^2)^2}\sin^2\phi_0 + \frac{3\pi\varepsilon_0^2}{4(1-\varepsilon_0^2)^{5/2}}\sin\phi_0\cos\phi_0 + \frac{2\varepsilon_0(1+\varepsilon_0^2)}{(1-\varepsilon_0^2)^3}\cos^2\phi_0\right] \quad (5)$$

Amortization of fluid film :

$$C = \frac{4}{W_r\lambda_k^2}\left[\frac{\pi}{2(1-\varepsilon_0^2)^{3/2}}\sin^2\phi_0 + \frac{4\varepsilon_0}{(1-\varepsilon_0^2)^2}\sin\phi_0\cos\phi_0 + \frac{\pi(1+2\varepsilon_0^2)}{2(1-\varepsilon_0^2)^{5/2}}\cos^2\phi_0\right] \quad (6)$$

With :

$$\frac{4}{W_r\lambda_k^2} = \frac{(1-\varepsilon_0^2)^2}{\varepsilon_0[16\varepsilon_0^2+\pi^2(1-\varepsilon_0^2)]^{1/2}}\cos^2\phi_0$$

$$\tan\phi_0 = \frac{\pi(1-\varepsilon_0^2)^{1/2}}{4\varepsilon_0}$$

$$\varepsilon_0 = 1-\frac{h}{c}$$

And:

λk defines the ratio of width length of contact area

$\varphi 0$ is the maximal angle, described by the distribution charge

Wr is the resultant force applied to fluid film

ε_0 is an eccentricity coefficient

c is the diametrical bearing clearance

h is the thickness of the fluid film

IV. RESULTS AND DISCUSSIONS

The general equation to solve a system of three degrees of freedom subjected to three forces, depending on the mass matrices, damping and stiffness being have a general form:

$$[A].\{\ddot{y}\}+[B].\{\dot{y}\}+[C].\{y\}=\{F\}$$

A, B and C represent respectively the masses of balls, damping and stiffness system.

$$A = \begin{bmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{bmatrix} \qquad B = \begin{bmatrix} C_{22} & -C_{22} & 0 \\ -C_{22} & C_{22}+C_{21} & -C_{21} \\ 0 & -C_{21} & C_{21} \end{bmatrix}$$

$$C = \begin{bmatrix} k_1 + k_{22} & -k_{22} & 0 \\ -k_{12} & k_{22} + k_{21} & -k_{21} \\ 0 & -k_{21} & k_3 + k_{21} \end{bmatrix}$$

The motion vector {y} and the force vector {f}. The vector F represents the impact forces generated by the fault of passage balls.

$$\{y\} = \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \end{Bmatrix} \qquad \{F\} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

The equation of motion, governing this system model is the following:

$$\begin{cases} M_1 \ddot{y}_1 + C_{22}\dot{y}_1 - C_{22}\dot{y}_2 + (k_1 + k_{22})y_1 - k_{22}y_2 = F_1 \\ M_2 \ddot{y}_2 - C_{22}\dot{y}_2 + (C_{22} + C_{21})\dot{y}_2 - C_{21}\dot{y}_3 - k_{22}y_1 + (k_{22} + k_{21})y_2 - k_{21}y_3 = F_2 \\ M_3 \ddot{y}_3 + C_{21}\dot{y}_3 - C_{21}\dot{y}_2 + (k_3 + k_{21})y_3 - k_{21}y_2 = F_3 \end{cases}$$

(7)

This can be written in matrix form:

$$\begin{bmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{bmatrix} \begin{Bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \\ \ddot{y}_3 \end{Bmatrix} + \begin{bmatrix} C_{22} & -C_{22} & 0 \\ -C_{22} & C_{22} + C_{21} & -C_{21} \\ 0 & -C_{21} & C_{21} \end{bmatrix} \begin{Bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \end{Bmatrix} + \begin{bmatrix} k_1 + k_{22} & -k_{22} & 0 \\ -k_{22} & k_{22} + k_{21} & -k_{21} \\ 0 & -C_{21} & k_3 + k_{21} \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

The trivial solution of the equation can be written as:

$$AY'' + BY' + CY = F$$

It becomes, after multiplication by A^{-1}:

$$Y'' + A^{-1}BY' + A^{-1}CY = A^{-1}F$$

We work in the case without default so F1 = F2 = F3 = 0,

Therefore $Y'' + A^{-1}BY' + A^{-1}CY = A^{-1}F$

A^{-1} is calculated by the shape:

$$A^{-1} = \frac{1}{\det A} coA^t$$

With:

$$\det(A) = M_1 M_2 M_3$$

And $A^{-1} = \begin{bmatrix} \frac{1}{M_1} & 0 & 0 \\ 0 & \frac{1}{M_2} & 0 \\ 0 & 0 & \frac{1}{M_3} \end{bmatrix}$

By replacing matrices by actual experimental values of the masses (M_1, M_2, M_3), damping (k_1, k_3, k_{21}, k_{23}) and stiffness (C_{21}, C_{23}).

k_1, k_2 calculated from (5)

C_{21}, C_{23}: calculated from (6)

$M_1 = 25*10^{-3}$ kg, $M_2 = 2*10^{-3}$ kg, $M_3 = 23*10^{-3}$ Kg

$k_1 = 10^7$ N/m, $k_3 = 10^7$ N/m, k_{21} = 0, 9925 N/m, k_{23} = 0, 9925 N/m

C_{21} = 9, 9072, C_{23} = 9, 9072

The mass matrix becomes:

$$B=\begin{pmatrix}0 & 0 & 0\\ 0 & 9,9 & -9,9\\ 0 & -9,9 & 9,9\end{pmatrix}$$

With:

$$A^{-1}B=\begin{pmatrix}40 & 0 & 0\\ 0 & 500 & 0\\ 0 & 0 & 43,5\end{pmatrix}\begin{pmatrix}0 & 0 & 0\\ 0 & 9,9 & -9,9\\ 0 & -9,9 & 9,9\end{pmatrix}$$

And:

$$A^{-1}B=\begin{pmatrix}0 & 0 & 0\\ 0 & 4950 & -4950\\ 0 & -413,25 & 413,25\end{pmatrix}$$

The damping matrix:

$$C=\begin{pmatrix}10^{7} & 0 & 0\\ 0 & 0,99 & -0,99\\ 0 & -0,99 & 10^{8}\end{pmatrix}$$

With:

$$A^{-1}C=\begin{pmatrix}40 & 0 & 0\\ 0 & 500 & 0\\ 0 & 0 & 43,5\end{pmatrix}\begin{pmatrix}10^{7} & 0 & 0\\ 0 & 0,99 & -0,99\\ 0 & -0,99 & 10^{8}\end{pmatrix}$$

And:

$$A^{-1}C=\begin{pmatrix}40.10^{7} & 0 & 0\\ 0 & 495 & -495\\ 0 & -43 & 435.10^{7}\end{pmatrix}$$

The matrix L which explicitly interpolating numerical application:

$$L=\begin{pmatrix}0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\\ 40.10^{7} & 0 & 0 & 0 & 0 & 0\\ 0 & 495 & -495 & 0 & 4950 & -4950\\ 0 & -43 & 435.10^{7} & 0 & -413,25 & 413,25\end{pmatrix}$$

In seeking solutions of X:

$$A-I_6X=\begin{pmatrix}-X & 0 & 0 & 1 & 0 & 0\\ 0 & -X & 0 & 0 & 1 & 0\\ 0 & 0 & -X & 0 & 0 & 1\\ 40.10^{7} & 0 & 0 & -X & 0 & 0\\ 0 & 495 & -495 & 0 & 4950-X & -4950\\ 0 & -43 & 435.10^{7} & 0 & -413,25 & 413,25-X\end{pmatrix}$$

The polynomial to be solved is given by:

$P(X) = X^6 - 5{,}363.X^5 - 4{,}75.10^9.X^4 + 2368.10^{13}.X^3 + 1{,}74.10^{18}.X^2 - 8{,}613.10^{21}.X - 8{,}61$

The own values are:

$$V_1 = 2.10^4$$

$$V_2 = -2.10^4$$

$$V_3 = -6{,}5763.10^4$$

$$V_4 = 6{,}6178.10^4$$

$$V_5 = 4{,}948.10^3$$

$$V_6 = -0{,}1$$

The vectors associated with each own values are:

$$\overrightarrow{V_1} = 5.10^{-5}\overrightarrow{e_1} + \overrightarrow{e_4}$$

$$\overrightarrow{V_2} = 5.10^{-5}\overrightarrow{e_1} - \overrightarrow{e_4}$$

$$\overrightarrow{V_3} = 1{,}0618.10^{-6}\overrightarrow{e_2} + 1{,}517.10^{-5}\overrightarrow{e_3} - 6{,}98.10^{-2}\overrightarrow{e_5} - 9{,}97.10^{-1}\overrightarrow{e_6}$$

$$\overrightarrow{V_4} = -1{,}22.10^{-6}\overrightarrow{e_2} - 1{,}5.10^{-5}\overrightarrow{e_3} + 8{,}06.10^{-2}\overrightarrow{e_5} - 9{,}97.10^{-1}\overrightarrow{e_6}$$

$$\overrightarrow{V_5} = 2{,}02.10^{-4}\overrightarrow{e_2} + 9{,}55.10^{-8}\overrightarrow{e_3} + \overrightarrow{e_5} + 4{,}72.10^{-4}\overrightarrow{e_6}$$

$$\overrightarrow{V_6} = -9{,}95.10^{-5}\overrightarrow{e_2} + 3{,}83.10^{-10}\overrightarrow{e_3} - 9{,}95.10^{-2}\overrightarrow{e_5} - 3{,}83.10^{-11}\overrightarrow{e_6}$$

The explicit matrix vectors calculated:

$$P = \begin{pmatrix} 5.10^{-5} & 5.10^{-5} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1{,}0618.10^{-6} & 1{,}22.10^{-6} & 2{,}02.10^{-4} & 9{,}95.10^{-1} \\ 0 & 0 & 1{,}517.10^{-5} & -1{,}5.10^{-5} & 9{,}55.10^{-8} & 3{,}83.10^{-10} \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -6{,}98.10^{-2} & 8{,}06.10^{-2} & 1 & -9{,}95.10^{-2} \\ 0 & 0 & -9{,}97.10^{-1} & -9{,}97.10^{-1} & 4{,}72.10^{-4} & -3{,}83.10^{-11} \end{pmatrix}$$

With:

P: matrix that contains the own vector

Furthermore P^{-1} results in the following matrix:

$$P^{-1} = \begin{pmatrix} 10^4 & 0 & 0 & 85.10^{-1} & 0 & 0 \\ 10^4 & 0 & 0 & -5.10^{-1} & 0 & 0 \\ 0 & -3{,}05.10^{-4} & 3{,}31.10^4 & 0 & -2{,}93.10^{-3} & -0{,}5 \\ 0 & -3{,}53.10^{-4} & -3{,}31.10^4 & 0 & 3{,}4.10^{-3} & -0{,}5 \\ 0 & 9{,}994.10^{-2} & 4{,}98.10^3 & 0 & 1 & 5{,}83.10^{-3} \\ 0 & 1 & -1 & 0 & -2{,}02.10^{-4} & -3{,}47.10^{-8} \end{pmatrix}$$

And:

$$P^{-1}AP=\begin{pmatrix} 2.10^4 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2.10^4 & 0 & 0 & 0 & 0 \\ 0 & 0 & -6,5763.10^4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6,6178.10^4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4,948.10^3 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0,1 \end{pmatrix}$$

And we consider :

$$\begin{pmatrix} y' \\ -y'' \end{pmatrix} = \begin{pmatrix} y_1^{'} \\ y_2^{'} \\ y_3^{'} \\ -y_4^{'} \\ -y_5^{'} \\ -y_6^{'} \end{pmatrix}$$

This vector can be written as follows:

$$\begin{pmatrix} y_1^{'} & 0 & 0 & 0 & 0 & 0 \\ 0 & y_2^{'} & 0 & 0 & 0 & 0 \\ 0 & 0 & y_3^{'} & 0 & 0 & 0 \\ 0 & 0 & 0 & -y_1^{''} & 0 & 0 \\ 0 & 0 & 0 & 0 & -y_2^{''} & 0 \\ 0 & 0 & 0 & 0 & 0 & -y_3^{''} \end{pmatrix} = e^{Vt} + K$$

With the vector V can be written as:

$$V=\begin{pmatrix} 0 & I \\ A^{-1}C & A^{-1}B \end{pmatrix}$$

The previous system is reflected in the resolution of the matrix:

$$\begin{pmatrix} y_1^{'} & 0 & 0 & 0 & 0 & 0 \\ 0 & y_2^{'} & 0 & 0 & 0 & 0 \\ 0 & 0 & y_3^{'} & 0 & 0 & 0 \\ 0 & 0 & 0 & -y_1^{''} & 0 & 0 \\ 0 & 0 & 0 & 0 & -y_2^{''} & 0 \\ 0 & 0 & 0 & 0 & 0 & -y_3^{''} \end{pmatrix} = \begin{pmatrix} e^{2.10^4 t}+1 & 0 & 0 & 0 & 0 & 0 \\ 0 & e^{-2.10^4 t}+1 & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{-6,5763.10^4 t}+1 & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{6,6178.10^4 t}+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{4,948.10^3 t}+1 & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{-0,1t}+1 \end{pmatrix}$$

Hence the withdraws:

$$y_1' = e^{2.10^4 t}+1$$

Incorporating includes:

$$y_1 = \frac{e^{2.10^4 t}}{2.10^4} + t + A$$

With the initial conditions imposed on the constant A to the value:

At t = 0 then y_1=0 $A=-\dfrac{1}{2.10^4}$

Hence the solution y_1:

$$y_1 = \frac{e^{2.10^4 t}}{2.10^4} + t - \frac{1}{2.10^4} \qquad (10)$$

Similarly for y1 we find the general expression of y_2:

$$y_2' = e^{-2.10^4 t} + 1$$

$$y_2 = -\frac{e^{-2.10^4 t}}{2.10^4} + t + B$$

With the initial conditions imposed there is the value of the constant B:

At t = 0 y2 =0 $B = \frac{1}{2.10^4}$

Where:

$$y_2 = -\frac{e^{-2.10^4 t}}{2.10^4} + t + \frac{1}{2.10^4} \qquad (11)$$

Integrating is the solution of the vector y_3:

$$y_3' = e^{-6,5763.10^4 t} + 1$$

$$y_3 = -\frac{e^{-6,5763.10^4 t}}{6,5763.10^4} + t + C$$

With the initial conditions imposed there is the value of the constant C:

At t = 0 $y_3 = 0$ $C = -\frac{1}{6,5763.10^4}$

$$y_3 = -\frac{e^{-6,5763.10^4 t}}{6,5763.10^4} + t - \frac{1}{6,5763.10^4}$$

V. CONCLUSION

Vibration analysis, for conditional preventive maintenance, proves a wonderful tool for several decades for the industry. It serves three levels of analysis: monitoring, diagnosis and damage of equipment state.

This work helped to develop an analytical model that simulates the behavior of a vibratory ball bearing with three degrees of freedom.The problem addressed in this paper is the development of an analytical simulator vibration bearing. Theoretical research is detailed enough to provide a simple model that lends itself easily to programming, but powerful enough to incorporate the effect of a maximum of influential parameters on the vibration of bearings.

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