

## The Effects of Soret and Dufour on an Unsteady MHD Free Convection Flow Past a Vertical Porous Plate In The Presence Of Suction or Injection

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**Abstract:** The objective of this paper is to analyze the effects of Soret and Dufour on an unsteady magneto hydrodynamic free convective fluid flow of a viscous incompressible and electrically conducting fluid past a vertical porous plate in the presence of suction or injection. The non – linear partial differential equations governing the flow have been solved numerically using finite difference method. The effects of the various parameters on the velocity, temperature and concentration profiles are presented graphically and values of skin – friction coefficient, Nusselt number and Sherwood number for various values of physical parameters are presented through tables.

**Keywords:** Soret number, Dufour number, Unsteady, MHD, Free convection flow, Vertical porous plate, Suction or Injection, Finite difference method.

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### I. NOMENCLATURE:

$B_o$	Magnetic field component along $y' -$ axis
$Gr$	Grashof number
$Gc$	Modified Grashof number
$g$	Acceleration of gravity
$K$	The permeability parameter
$M$	Hartmann number
$Pr$	Prandtl number
$Sc$	Schmidt number
$Sr$	Soret number
$Du$	Dufour number
$D$	Chemical molecular diffusivity
$T'$	Temperature of fluid near the plate
$T'_w$	Temperature of the fluid far away of the fluid from the plate
$T_m$	Mean fluid temperature
$T'_\infty$	Temperature of the fluid at infinity
$C$	Concentration of the fluid
$c_s$	Concentration susceptibility
$C'$	Concentration of fluid near the plate
$C'_w$	Concentration of the fluid far away of the fluid from the plate
$C'_\infty$	Concentration of the fluid at infinity
$t'$	Time in $x', y'$ coordinate system
$k_T$	Thermal diffusion ratio
$t$	Time in dimensionless co – ordinates
$u'$	Velocity component in $x' -$ direction

$v'$	Velocity component in $y'$ – direction
$u$	Dimensionless velocity component in $x'$ – direction
$Nu$	Nusselt number
$Sh$	Sherwood number
$k_T$	Thermal conductivity of the fluid
$R_{e_x}$	Reynold's number
$c_p$	Specific heat at constant pressure
$D_m$	Mass diffusivity
$x', y'$	Co – ordinate system
$x, y$	Dimensionless coordinates
$v_o$	Suction or Injection parameter

**Greek symbols:**

$\beta$	Coefficient of volume expansion for heat transfer
$\beta^*$	Coefficient of volume expansion for mass transfer
$\kappa$	Thermal conductivity, W/mK
$\sigma$	Electrical conductivity of the fluid
$\nu$	Kinematic viscosity
$\theta$	Non – dimensional temperature
$\rho$	Density of the fluid
$\tau$	Skin – friction

**II. INTRODUCTION:**

The subject of convective flow in porous media has attracted considerable attention in the last several decades and is now considered to be an important field of study in the general areas of fluid dynamics and heat transfer. This topic has important applications, such as heat transfer associated with heat recovery from geothermal systems and particularly in the field of large storage systems of agricultural products, heat transfer associated with storage of nuclear waste, exothermic reaction in packedbed reactors, heat removal from nuclear fuel debris, flows in soils, petroleum extraction, control of pollutant spread in groundwater, solar power collectors and porous material regenerative heat exchangers, to name just a few applications. The range of free convective flows that occur in nature and in engineering practice is very large and have been extensively considered by many researchers. When heat and mass transfer occur simultaneously in a moving fluid, the relations between the fluxes and the driving potentials may be of a more intricate nature. An energy flux can be generated not only by temperature gradients but by composition gradients also. The energy flux caused by a composition gradient is termed the Dufour or diffusion – thermo effect. On the other hand, mass fluxes can also be created by temperature gradients and this embodies the Soret or thermal – diffusion effect. Such effects are significant when density differences exist in the flow regime. For example, when species are introduced at a surface in a fluid domain, with a different (lower) density than the surrounding fluid, both Soret (thermo – diffusion) and Dufour (diffusion – thermo) effects can become influential. Soret and Dufour effects are important for intermediate molecular weight gases in coupled heat and mass transfer in fluid binary systems, often encountered in chemical process engineering and also in high – speed aerodynamics. Generally, the thermal – diffusion and the diffusion – thermo effects are of smaller – order magnitude than the effects prescribed by Fourier's or Fick's laws and are often neglected in heat and mass transfer processes. However, there are exceptions. The thermal – diffusion effect, for instance, has been utilized for isotope separation and in mixture between gases with very light molecular weight (Hydrogen – Helium) and of medium molecular weight (Nitrogen – Air) the diffusion – thermo effect was found to be of a magnitude such that it cannot be neglected.

Abdul Maleque *et al.* [1] studied the effects of variable properties and Hall current on steady MHD laminar convective fluid flow due to a porous rotating disk. Abreu *et al.* [2] examined the boundary layer solutions for the cases of forced, natural and mixed convection under a continuous set of similarity type variables determined by a combination of pertinent variables measuring the relative importance of buoyancy force term in the momentum equation. Afify [3] carried out an analysis to study free convective heat and mass transfer of an incompressible, electrically conducting fluid over a stretching sheet in the presence of suction and injection with thermal – diffusion and diffusion – thermo effects. Alam *et al.* [4] studied numerically the Dufour

and Soret effects on combined free – forced convection and mass transfer flow past a semi – infinite vertical plate under the influence of transversely applied magnetic field. Ambethkar [5] investigated numerical solutions of heat and mass transfer effects of an unsteady MHD free convective flow past an infinite vertical plate with constant suction. Anand Rao and Srinivasa Raju [6] investigated applied magnetic field on transient free convective flow of an incompressible viscous dissipative fluid in a vertical channel. Anjali Devi *et al.* [7] discussed pulsated convective MHD flow with Hall current, heat source and viscous dissipation along a vertical porous plate. Atul Kumar Singh *et al.* [8] studied hydromagnetic free convection and mass transfer flow with Joule heating, thermal diffusion, Heat source and Hall current. Chaudhary *et al.* [9] studied heat and mass transfer in elasticoviscous fluid past an impulsively started infinite vertical plate with Hall Effect. Chaudhary *et al.* [10] analyzed Hall Effect on MHD mixed convection flow of a Viscoelastic fluid past an infinite vertical porous plate with mass transfer and radiation. Chin *et al.* [11] obtained numerical results for the steady mixed convection boundary layer flow over a vertical impermeable surface embedded in a porous medium when the viscosity of the fluid varies inversely as a linear function of the temperature. On the effectiveness of viscous dissipation and Joule heating on steady MHD and slip flow of a bingham fluid over a porous rotating disk in the presence of Hall and ion – slip currents was studied by Emmanuel Osalusi *et al.* [12].

Gaikwad *et al.* [13] investigated the onset of double diffusive convection in two component couple of stress fluid layer with Soret and Dufour effects using both linear and non – linear stability analysis. Hayat *et al.* [14] discussed the effects of Soret and Dufour on heat and mass transfer on mixed convection boundary layer flow over a stretching vertical surface in a porous medium filled with a viscoelastic fluid, mixed convection flow and heat transfer over a porous stretching surface in porous medium. Osalusi *et al.* [15] investigated thermo – diffusion and diffusion – thermo effects on combined heat and mass transfer of a steady hydromagnetic convective and slip flow due to a rotating disk in the presence of viscous dissipation and ohmic heating. Pal *et al.* [16] analyzed the combined effects of mixed convection with thermal radiation and chemical reaction on MHD flow of viscous and electrically conducting fluid past a vertical permeable surface embedded in a porous medium. Shateyi [17] investigated thermal radiation and buoyancy effects on heat and mass transfer over a semi – infinite stretching surface with suction and blowing. More recently, Shivaiah and Anand Rao [18] was analyzed the effect of chemical reaction on unsteady magnetohydrodynamic free convective fluid flow past a vertical porous plate in the presence of suction or injection. Srihari *et al.* [19] discussed Soret effect on unsteady MHD free convective mass transfer flow past an infinite vertical porous plate with oscillatory suction velocity and heat sink. Sriramulu *et al.* [20] discussed the effect of Hall Current on MHD flow and heat transfer along a porous flat plate with mass transfer. Vempati *et al.* [21] studied numerically the effects of Dufour and Soret numbers on an unsteady MHD flow past an infinite vertical porous plate with thermal radiation.

Motivated by the above reference work and the numerous possible industrial applications of the problem (like in isotope separation), it is of paramount interest in this study to investigate the effects of Soret and Dufour on magnetohydrodynamic flow along a vertical porous plate in presence of suction or Injection. The governing equations are transformed by using unsteady similarity transformation and the resultant dimensionless equations are solved by using the finite difference method. The effects of various governing parameters on the velocity, temperature, concentration, skin – friction coefficient, Nusselt number and Sherwood number are shown in figures and tables and discussed in detail. From computational point of view it is identified and proved beyond all doubts that the finite difference method is more economical in arriving at the solution and the results obtained are good agreement with the results of Shivaiah and Anand Rao [18] in some special cases.

### III. MATHEMATICAL FORMULATION:

The effects of Soret and Dufour on an unsteady two – dimensional magnetohydrodynamic free convection flow of a viscous incompressible and electrically conducting fluid past a vertical porous plate in the presence of suction or injection is considered.

We made the following assumptions.

1. In Cartesian coordinate system, let  $x'$  – axis is taken to be along the plate and the  $y'$  – axis normal to the plate. Since the plate is considered infinite in  $x'$  – direction, hence all physical quantities will be independent of  $x'$  – direction.
2. Let the components of velocity along  $x'$  and  $y'$  axes be  $u'$  and  $v'$  which are chosen in the upward direction along the plate and normal to the plate respectively.

3. Initially, the plate and the fluid are at the same temperature  $T'_\infty$  and the concentration  $C'_\infty$ . At a time  $t' > 0$ , the plate temperature and concentration are raised to  $T'_w$  and  $C'_w$  respectively and are maintained constantly thereafter.
4. A uniform magnetic field of magnitude  $B_o$  is applied in the direction perpendicular to the plate. The fluid is assumed to be slightly conducting, and hence the magnetic Reynolds number is much less than unity and the induced magnetic field is negligible in comparison with the applied magnetic field.
5. It is assumed that the external electric field is zero and the electric field due to the polarization of charges is negligible.
6. The homogeneous chemical reaction of first order with rate constant between the diffusing species and the fluid is neglected.
7. The Hall effect of magnetohydrodynamics and magnetic dissipation (Joule heating of the fluid) are neglected.
8. It is also assumed that all the fluid properties are constant except that of the influence of the density variation with temperature and concentration in the body force term (Boussinesq's approximation).  
Under these assumptions, the governing boundary layer equations of the flow field are:

Continuity Equation:

$$\frac{\partial v'}{\partial y'} = 0$$

(1)

Momentum Equation:

$$\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} = g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) + \nu \frac{\partial^2 u'}{\partial y'^2} - \left( \frac{\sigma B_o^2}{\rho} + \frac{\nu}{K'} \right) u' \quad (2)$$

Energy Equation:

$$\frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} = \frac{\kappa}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} + \frac{D_m k_T}{c_s c_p} \frac{\partial^2 C'}{\partial y'^2} \quad (3)$$

Species Diffusion Equation:

$$\frac{\partial C'}{\partial t'} + v' \frac{\partial C'}{\partial y'} = D \frac{\partial^2 C'}{\partial y'^2} + \frac{D_m k_T}{T_m} \frac{\partial^2 T'}{\partial y'^2} \quad (4)$$

And the corresponding boundary conditions are

$$\left. \begin{aligned} t' \leq 0 : u' = 0, v' = 0, T' = T'_\infty, C' = C'_\infty \text{ for all } y' \\ t' > 0 : \left\{ \begin{aligned} u' = 0, v' = v(t), T' = T'_w, C' = C'_w \text{ at } y' = 0 \\ u' \rightarrow 0, v' \rightarrow 0, T' \rightarrow T'_\infty, C' \rightarrow C'_\infty \text{ as } y' \rightarrow \infty \end{aligned} \right\} \end{aligned} \right\} \quad (5)$$

From the continuity equation, it can be seen that  $v'$  is either a constant or a function of time. So, assuming suction velocity is to be oscillatory about a non – zero constant mean, one can write  $v' = -v_o$

(6)

The negative sign indicates that the suction velocity is directed towards the plate. In order to write the governing equations and the boundary condition in dimension less form, the following non – dimensional quantities are introduced.

$$\left. \begin{aligned} y = \frac{y' V_o}{\nu}, t = \frac{t' V_o^2}{\nu}, u = \frac{u'}{V_o}, v = \frac{v'}{V_o}, \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \\ Gr = \frac{g\beta\nu(T'_w - T'_\infty)}{V_o^3}, Sc = \frac{\nu}{D}, Gc = \frac{g\beta^*\nu(C'_w - C'_\infty)}{V_o^3}, Pr = \frac{\nu\rho c_p}{\kappa}, \\ M = \left( \frac{\sigma B_o^2}{\rho} \right) \frac{\nu}{V_o^2}, K = \frac{K' V_o^2}{\nu^2}, Du = \frac{D_m k_T (C'_w - C'_\infty)}{c_s c_p (T'_w - T'_\infty)}, Sr = \frac{D_m k_T (T'_w - T'_\infty)}{\nu T_m (C'_w - C'_\infty)} \end{aligned} \right\} \quad (7)$$

Hence, using the above non – dimensional quantities, the equations (2) – (5) in the non – dimensional form can be written as

$$\frac{\partial u}{\partial t} - v_o \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + (Gr)\theta + (Gc)C - \left( M + \frac{1}{K} \right) u \quad (8)$$

$$\frac{\partial \theta}{\partial t} - v_o \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} + Du \left( \frac{\partial^2 C}{\partial y^2} \right)$$

$$(9) \frac{\partial C}{\partial t} - v_o \frac{\partial C}{\partial y} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} + Sr \left( \frac{\partial^2 \theta}{\partial y^2} \right)$$

(10)

And the corresponding boundary conditions are

$$\left. \begin{aligned} t \leq 0: & \quad u = 0, \theta = 0, C = 0 \text{ for all } y \\ t > 0: & \quad \left\{ \begin{aligned} u = 0, \theta = 1, C = 1 \text{ at } y = 0 \\ u \rightarrow 0, \theta \rightarrow 0, C \rightarrow 0 \text{ as } y \rightarrow \infty \end{aligned} \right\} \end{aligned} \right\} \quad (11)$$

All the physical parameters are defined in the nomenclature.

It is now important to calculate the physical quantities of primary interest, which are the local wall shear stress, the local surface heat and mass flux. Given the velocity field in the boundary layer, we can now calculate the local wall shear stress (i.e., skin – friction) is given by and in dimensionless form, we obtain Knowing the temperature field, it is interesting to study the effect of the free convection and radiation on the rate of heat transfer. This is given by which is written in dimensionless form as

$$\tau = \frac{\tau_w}{\rho u_w^2}, \tau_w = \left[ \mu \frac{\partial u}{\partial y} \right]_{y'=0} = \rho v_o^2 u'(0) = \left[ \frac{\partial u}{\partial y} \right]_{y=0} \quad (12)$$

The dimensionless local surface heat flux (i.e., Nusselt number) is obtained as

$$Nu_u(x') = - \left[ \frac{x'}{(T'_w - T'_\infty)} \frac{\partial T'}{\partial y'} \right]_{y'=0} \quad \text{then } Nu = \frac{Nu_u(x')}{R_{e_x}} = - \left[ \frac{\partial \theta}{\partial y} \right]_{y=0} \quad (13)$$

The definition of the local mass flux and the local Sherwood number are respectively given by with the help of these equations, one can write

$$Sh_h(x') = - \left[ \frac{x'}{(C'_w - C'_\infty)} \frac{\partial C'}{\partial y'} \right]_{y'=0} \quad \text{then } Sh = \frac{Sh_h(x')}{R_{e_x}} = - \left[ \frac{\partial C}{\partial y} \right]_{y=0} \quad (14)$$

Where  $R_{e_x} = - \frac{v_o x'}{\nu}$  is the Reynold's number.

#### IV. METHOD OF SOLUTION:

Equations (8) – (10) are coupled non – linear partial differential equations and are solved by using initial and boundary conditions (11). However, exact or approximate solutions are not possible for this set of equations. And hence we solve these equations by an implicit finite difference method of Crank – Nicolson type for a numerical solution. The equivalent finite difference scheme of equations (8) – (10) is as follows:

$$\left( \frac{u_{i,j+1} - u_{i,j}}{\Delta t} \right) - v_o \left( \frac{u_{i+1,j} - u_{i,j}}{\Delta y} \right) = \frac{1}{2} \left( \frac{u_{i-1,j+1} - 2u_{i,j+1} + u_{i+1,j+1}}{(\Delta y)^2} + \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{(\Delta y)^2} \right)$$

$$- \frac{1}{2} \left( M + \frac{1}{K} \right) \left( \frac{u_{i,j+1} + u_{i,j}}{\Delta y} \right) + \frac{1}{2} (Gr) \left( \frac{\theta_{i,j+1} + \theta_{i,j}}{\Delta y} \right) + \frac{1}{2} (Gc) \left( \frac{C_{i,j+1} + C_{i,j}}{\Delta y} \right)$$

$$(15) \left( \frac{\theta_{i,j+1} - \theta_{i,j}}{\Delta t} \right) - v_o \left( \frac{\theta_{i+1,j} - \theta_{i,j}}{\Delta y} \right) = \frac{1}{2Pr} \left( \frac{\theta_{i-1,j+1} - 2\theta_{i,j+1} + \theta_{i+1,j+1}}{(\Delta y)^2} + \frac{\theta_{i-1,j} - 2\theta_{i,j} + \theta_{i+1,j}}{(\Delta y)^2} \right) +$$

$$\frac{Du}{2} \left( \frac{C_{i-1,j+1} - 2C_{i,j+1} + C_{i+1,j+1}}{(\Delta y)^2} + \frac{C_{i-1,j} - 2C_{i,j} + C_{i+1,j}}{(\Delta y)^2} \right)$$

(16)

$$\left( \frac{C_{i,j+1} - C_{i,j}}{\Delta t} \right) - v_o \left( \frac{C_{i+1,j} - C_{i,j}}{\Delta y} \right) = \frac{1}{2Sc} \left( \frac{u_{i-1,j+1} - 2u_{i,j+1} + u_{i+1,j+1}}{(\Delta y)^2} + \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{(\Delta y)^2} \right)$$

$$+ \frac{Sr}{2} \left( \frac{\theta_{i-1,j+1} - 2\theta_{i,j+1} + \theta_{i+1,j+1}}{(\Delta y)^2} + \frac{\theta_{i-1,j} - 2\theta_{i,j} + \theta_{i+1,j}}{(\Delta y)^2} \right)$$

(17) Here the suffix  $i$  corresponds to  $y$  and  $j$  corresponds to  $t$ .

Also  $\Delta t = t_{j+1} - t_j$  and  $\Delta y = y_{i+1} - y_i$

The complete solution of the discrete equations (15) – (17) proceeds as follows:

**Step – (1):** Knowing the values of  $C$ ,  $\theta$  and  $u$  at a time  $t = j$ , calculate  $C$  and  $\theta$  at a time  $t = j + 1$  using equations (16) and (17) and solving tri – diagonal linear system of equations.

**Step – (2):** Knowing the values of  $\theta$  and  $C$  at time  $t = j$ , solve the equation (15) (via tri – diagonal matrix inversion) to obtain  $u$  at a time  $t = j + 1$ .

We can repeat steps (1) and (2) to proceed from  $t = 0$  to the desired time value.

The implicit Crank – Nicolson method is a second order method ( $O(\Delta t^2)$ ) in time and has no restrictions on space and time – steps,  $\Delta y$  and  $\Delta t$ , *i.e.*, the method is unconditionally stable. The finite differences scheme used, involves the values of the function at the six grid points. A linear combination of the “future” points is equal to another linear combination of the “present” points. To find the future values of the function, one must solve a system of linear equations, whose matrix has a tri – diagonal form. The computations were carried out for  $Gr = 1.0$ ,  $Gc = 1.0$ ,  $Pr = 0.71$  (Air),  $Sc = 0.22$  (Hydrogen),  $M = 1.0$ ,  $K = 1.0$ ,  $v_o = 1.0$ ,  $Sr = 1.0$ ,  $Du = 1.0$  and  $\Delta y = 0.1$ ,  $\Delta t = 0.001$  and the procedure is repeated till  $y = 4$ . In order to check the accuracy of numerical results, the present study is compared with the available theoretical solution of Shivaiah and Anand Rao [18] and they are found to be in good agreement.

## V. RESULTS AND DISCUSSION:

The effects of Soret and Dufour on an unsteady magnetohydrodynamic free convective fluid flow past a vertical porous plate in the presence of suction or injection have been studied. The governing equations are solved by using the finite difference method and approximate solutions are obtained for velocity field, temperature field, concentration distribution, skin – friction coefficient, Nusselt number and Sherwood number. The effects of the pertinent parameters on the flow field are analyzed and discussed with the help of velocity profiles figures (1) – (9), temperature profiles figures (10) – (12), concentration distribution figures (13) – (15) and tables (1) and (2).

### 4.1 Velocity field:

The velocity of the flow field is found to change more or less with the variation of the flow parameters. The major factors affecting the velocity the flow field are the thermal Grashof number  $Gr$ , solutal Grashof number  $Gc$ , Hartmann number  $M$ , Permeability parameter  $K$ , Prandtl number  $Pr$ , Schmidt number  $Sc$ , Soret number  $Sr$ , Dufour number  $Du$  and Suction or Injection parameter  $v_o$ . The effects of these parameters on the velocity field have been analyzed with the help of figures (1) – (9).

#### 4.1.1 Effect of thermal Grashof number ( $Gr$ ):

The influence of the thermal Grashof number on the velocity is presented in figure (1). The thermal Grashof number signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. As expected, it is observed that there is a rise in the velocity due to the enhancement of thermal buoyancy force. Here, the positive values of  $Gr$  correspond to cooling of the plate. Also, as  $Gr$  increases, the peak values of the velocity increases rapidly near the porous plate and then decays smoothly to the free stream velocity.

#### 4.1.2 Effect of solutal Grashof number ( $Gc$ ):

Figure (2) presents typical velocity profiles in the boundary layer for various values of the solutal Grashof number  $Gc$ , while all other parameters are kept at some fixed values. The solutal Grashof number  $Gc$  defines the ratio of the species buoyancy force to the viscous hydrodynamic force. As expected, the fluid velocity increases and the peak value is more distinctive due to increase in the species buoyancy force. The velocity distribution attains a distinctive maximum value in the vicinity of the plate and then decreases properly to approach the free stream value.

#### 4.1.3 Effect of Prandtl number ( $Pr$ ):

Figure (3) shows the behavior velocity for different values Prandtl number  $Pr$ . The numerical results show that the effect of increasing values of Prandtl number results in a decreasing velocity. It is observed that an increase in the Prandtl number results a decrease of the thermal boundary layer thickness within the boundary layer.

#### 4.1.4 Effect of Schmidt number ( $Sc$ ):

The nature of velocity profiles in presence of foreign species such as Hydrogen ( $Sc = 0.22$ ), Helium ( $Sc = 0.30$ ), Water – vapour ( $Sc = 0.60$ ) and Oxygen ( $Sc = 0.66$ ) are shown in the figure (4). The flow field suffers a decrease in velocity at all points in presence of heavier diffusing species.

#### 4.1.5 Effect of Hartmann number ( $M$ ):

The effect of the Hartmann number ( $M$ ) is shown in the figure (5). It is observed that the velocity of the fluid decreases with the increase of the Hartmann number values, because the presence of a magnetic field in an electrically conducting fluid introduces a force called the Lorentz force, which acts against the flow if the magnetic field is applied in the normal direction.

#### 4.1.6 Effect of Permeability parameter ( $K$ ):

Figure (6) shows the effect of the permeability parameter  $K$  on the velocity distribution. The velocity field is increasing with the increasing dimensionless porous medium parameter. Physically, this result can be achieved when the holes of the porous medium may be neglected.

#### 4.1.7 Effect of Soret number ( $Sr$ ):

Figure (7) shows the effect of Soret number  $Sr$  on the velocity distribution. We observe that the velocity increases with the increase of Soret number.

#### 4.1.8 Effect of Dufour number ( $Du$ ):

The effect of Dufour number  $Du$  on velocity distribution is as shown in the figure (8). From this figure (8) we observe that the velocity is increases with increasing values of Dufour number  $Du$ .

#### 4.1.9 Effect of Suction or Injection parameter ( $v_o$ ):

Figure (9), we present the variation in the velocity of the flow field due to the change of the suction or injection keeping other parameters of the flow field constant. It is observed that suction or injection parameter retards the velocity of flow field at all points. As the suction or injection of the fluid through the plate increases the plate is cooled down and in consequence of which the viscosity of the flowing fluid increases. Therefore, there is a gradual decrease in the velocity of the fluid as  $v_o$  increases.

### 4.2 Temperature field:

The temperature of the flow suffers a substantial change with the variation of the flow parameters such as Prandtl number  $Pr$ , Dufour number  $Du$ , Suction or Injection parameter  $v_o$ , these variations are shown in figures (10) – (12).

#### 4.2.1 Effect of Prandtl number ( $Pr$ ):

From figure (10) depicts the effect of Prandtl number against  $y$  on the temperature field keeping other parameters of the flow field constant. It is interesting to observe that an increase in the Prandtl number  $Pr$  decreases the temperature of the flow field.

#### 4.2.2 Effect of Dufour number ( $Du$ ):

The effect of Dufour number  $Du$  on temperature field against  $y$  is as shown in the figure (11). From this figure, it is observe that an increase in the Dufour number  $Du$  increases the temperature of the flow field.

#### 4.2.3 Effect of suction or injection parameter ( $v_o$ ):

The effect of suction or injection parameter on the temperature of the flow field is shown in figure (12). The temperature of the flow field is found to decrease in the presence of the growing suction or injection. The temperature profile becomes very much linear in absence of suction or injection  $v_o$ . In presence of higher suction or injection more amount of fluid is pushed into the flow field through the plate due to which the flow field suffers a decrease in temperature at all points.

**4.3 Concentration distribution:**

The variation in the concentration boundary layer of the flow field with the flow parameters Schmidt number  $Sc$ , Chemical reaction parameter  $Kr$  and Suction or Injection parameter ( $v_0$ ) are shown in figures 13,14 and 15.

**4.3.1 Effect of Schmidt number ( $Sc$ ):**

The concentration distribution is vastly affected by the presence of foreign species such as Hydrogen ( $Sc = 0.22$ ), Helium ( $Sc = 0.30$ ), Water – vapour ( $Sc = 0.60$ ) and Oxygen ( $Sc = 0.66$ ) are shown in figure (13). From this figure (13), we observe that the effect of  $Sc$  on the concentration distribution of the flow field. The concentration distribution is found to decrease faster as the diffusing foreign species becomes heavier. Thus higher  $Sc$  leads to a faster decrease in concentration of the flow field.

**4.3.2 Effect of Soret number ( $Sr$ ):**

Figure (14) shows the effect of Soret number  $Sr$  on the concentration distribution. We observe that the concentration increases with the increase of Soret number.

**4.3.3 Effect of Suction or Injection parameter ( $v_o$ ):**

Figure (15), depicts the concentration profiles against  $y$  for various values of suction parameter  $v_o$  keeping other parameters are constant. Suction parameter is found to decrease the concentration of the flow field at all points. In other words, cooling of the plate is faster as the suction parameter becomes larger. Thus it may be concluded that larger suction leads to faster cooling of the plate.

**Table – 1:** Skin – friction results ( $\tau$ ) for the values of  $Gr, Gc, Pr, Sc, M, K, Sr, Du$  and  $v_o$

$Gr$	$Gc$	$Pr$	$Sc$	$M$	$K$	$Sr$	$Du$	$v_o$	$\tau$
1.0	1.0	0.71	0.22	1.0	1.0	1.0	1.0	1.0	4.6976
2.0	1.0	0.71	0.22	1.0	1.0	1.0	1.0	1.0	5.3053
1.0	2.0	0.71	0.22	1.0	1.0	1.0	1.0	1.0	6.2565
1.0	1.0	7.00	0.22	1.0	1.0	1.0	1.0	1.0	3.4661
1.0	1.0	0.71	0.30	1.0	1.0	1.0	1.0	1.0	4.1087
1.0	1.0	0.71	0.22	2.0	1.0	1.0	1.0	1.0	4.5630
1.0	1.0	0.71	0.22	1.0	2.0	1.0	1.0	1.0	5.0069
1.0	1.0	0.71	0.22	1.0	1.0	2.0	1.0	1.0	4.8875
1.0	1.0	0.71	0.22	1.0	1.0	1.0	2.0	1.0	5.0023
1.0	1.0	0.71	0.22	1.0	1.0	1.0	1.0	2.0	4.1236

**4.4 Skin – friction coefficient ( $\tau$ ):**

Tables – 1 show numerical values of the Skin – friction coefficient ( $\tau$ ) for various values of thermal Grashof number  $Gr$ , solutal Grashof number  $Gc$ , Hartmann number  $M$ , Permeability parameter  $K$ , Prandtl number  $Pr$ , Soret number  $Sr$ , Schmidt number  $Sc$ , Dufour number  $Du$  and Suction or Injection parameter  $v_o$ . From table – 1, we observed that, an increase in the Hartmann number, Prandtl number, Schmidt number and Suction or Injection parameter decrease in the value of the skin – friction coefficient while an increase in the thermal Grashof number, solutal Grashof number, Permeability parameter, Soret number and Dufour number increase in the value of the skin – friction coefficient.

**Table – 2:** Rate of heat transfer ( $Nu$ ) values for different values of  $Pr, Du$  and  $v_o$  and Rate of mass transfer ( $Sh$ ) values for different values of  $Sc, Sr$  and  $v_o$

$Pr$	$Du$	$Nu$	$Sc$	$k_r$	$Sh$
0.71	1.0	1.1521	0.22	1.0	2.2148
7.00	1.0	0.6987	0.30	1.0	1.1175
0.71	2.0	1.2750	0.22	2.0	2.3474



**4.5 Heat transfer coefficient or Nusselt number ( $Nu$ ):**

Table – 2 show the numerical values of heat transfer coefficient in terms of Nusselt number ( $Nu$ ) for various values of Prandtl number  $Pr$ , Dufour number  $Du$  and Suction or Injection parameter  $v_o$ . It is observed that, an increase in the Prandtl number or Suction or Injection parameter decrease in the value of heat transfer coefficient while an increase in the Dufour number increase in the value of heat transfer coefficient.

**4.6 Mass transfer coefficient or Sherwood number ( $Sh$ ):**

Table – 2 show the numerical values of mass transfer coefficient in terms of Sherwood number ( $Sh$ ) for various values of Schmidt number  $Sc$ , Soret number  $Sr$  and Suction or Injection parameter  $v_o$ . It is observed that, an increase in the Schmidt number, Suction or Injection parameter decreases in the value of mass transfer coefficient and an increase in the Soret number increases in the value of mass transfer coefficient.

**Table – 3:** Comparison of present Skin – friction results ( $\tau$ ) with the Skin – friction results ( $\tau^*$ ) obtained by Shivaiah and Anand Rao [18] for different values of  $Gr, Gc, Pr, Sc, M, K$  and  $v_o$

$Gr$	$Gc$	$Pr$	$Sc$	$M$	$K$	$v_o$	$\tau$	$\tau^*$
1.0	1.0	0.71	0.22	1.0	1.0	1.0	1.6554	1.6541
2.0	1.0	0.71	0.22	1.0	1.0	1.0	2.1067	2.1054
1.0	2.0	0.71	0.22	1.0	1.0	1.0	2.5162	2.5149
1.0	1.0	7.00	0.22	1.0	1.0	1.0	1.2344	1.2337
1.0	1.0	0.71	0.30	1.0	1.0	1.0	1.4170	1.4163
1.0	1.0	0.71	0.22	2.0	1.0	1.0	1.3322	1.3315
1.0	1.0	0.71	0.22	1.0	2.0	1.0	2.2181	2.2166
1.0	1.0	0.71	0.22	1.0	1.0	2.0	1.2137	1.2128

In order to ascertain the accuracy of the numerical results, the present results are compared with the previous results of Shivaiah and Anand Rao [18] for  $Gr = Gc = 1.0$ ,  $Pr = 0.71$ ,  $Sc = 0.22$ ,  $M = 1.0$ ,  $K = 1.0$  and  $v_o = 1.0$  in table – 3. They are found to be in an excellent agreement.

**VI. CONCLUSIONS:**

In this study, we examined the effects of Soret and Dufour numbers on an unsteady magnetohydrodynamic free convective fluid flow past a vertical porous plate in the presence of suction or injection. The leading governing equations are solved numerically by employing the highly efficient finite difference method. We present results to illustrate the flow characteristics for the velocity, temperature, concentration, skin – friction coefficient, Nusselt number and Sherwood number and show how the flow fields are influenced by the material parameters of the flow problem. We can conclude from these results that

- [1] An increase in  $Pr, Sc, M$  and  $v_o$  decrease the velocity field, while an increase in  $Gr, Gc, K, Sr$  and  $Du$  decrease the velocity field.
- [2] An increase in  $Du$  increases the temperature distribution, while an increase in  $Pr$  and  $v_o$  decrease the temperature distribution.
- [3] An increase in  $Sr$  increases the concentration distribution, while an increase in  $Sc$  and  $v_o$  decrease the concentration distribution.
- [4] An increase in  $Pr, Sc, M$  and  $v_o$  decrease the skin – friction coefficient, while an increase in  $Gr, Gc, K, Sr$  and  $Du$  decrease the skin – friction coefficient.
- [5] An increase in  $Du$  increases the heat transfer coefficient, while an increase in  $Pr$  and  $v_o$  decrease the heat transfer coefficient.
- [6] An increase in  $Sr$  increases the mass transfer coefficient, while an increase in  $Sc$  and  $v_o$  decrease the mass transfer coefficient.
- [7] On comparing the skin – friction ( $\tau$ ) results with the skin – friction ( $\tau^*$ ) results of Shivaiah and Anand Rao [18] it can be seen that they agree very well.

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Graphs:

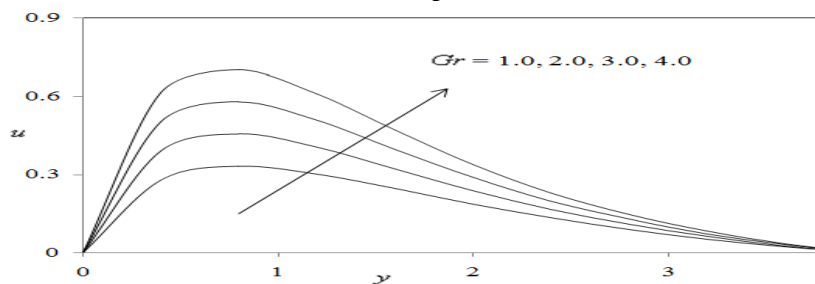


Figure 1. Velocity profiles for different values of  $Gr$

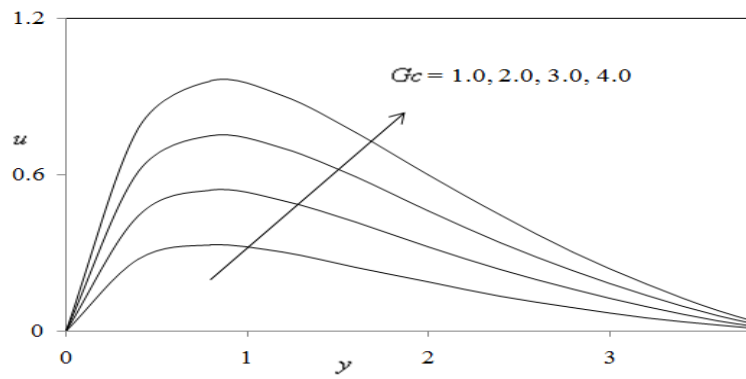


Figure 2. Velocity profiles for different values of  $Gc$

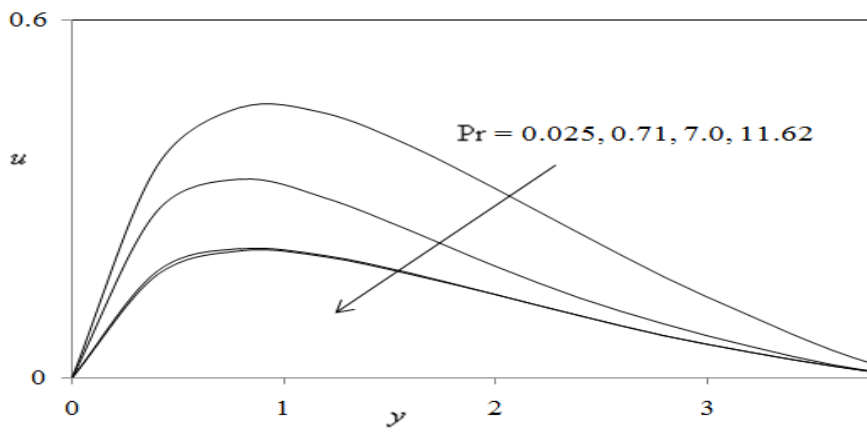


Figure 3. Velocity profiles for different values of  $Pr$

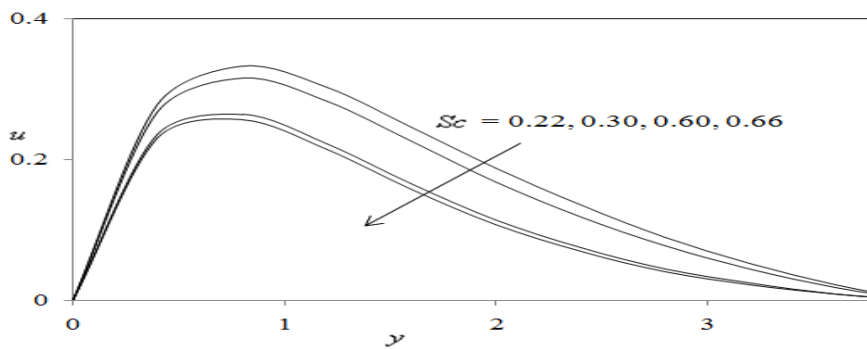


Figure 4. Velocity profiles for different values of  $Sc$

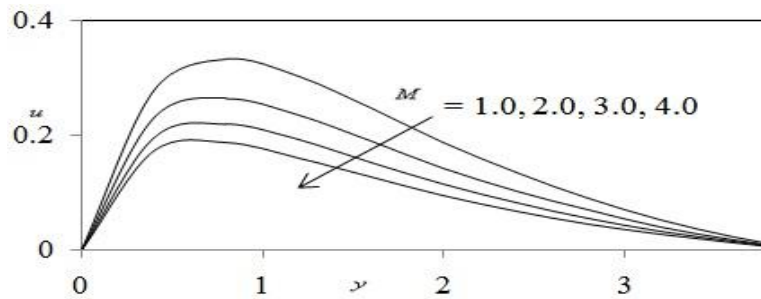


Figure 5. Velocity profiles for different values of  $M$

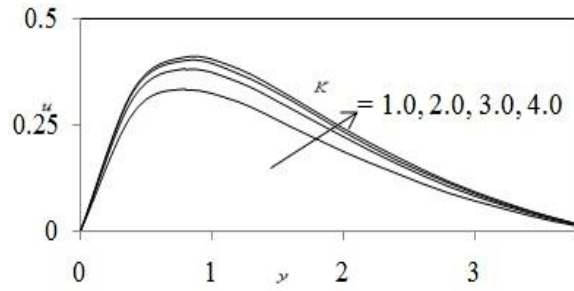


Figure 6. Velocity profiles for different values of  $K$

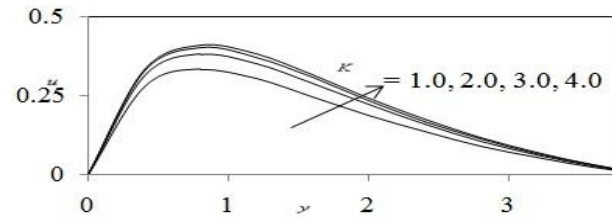


Figure 7. Velocity profiles for different values of  $Sr$

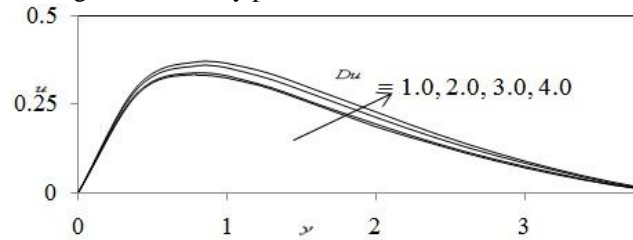


Figure 8. Velocity profiles for different values of  $Du$

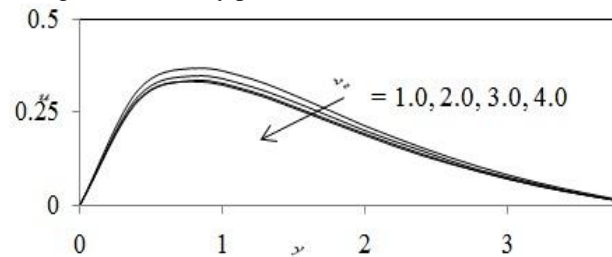


Figure 9. Velocity profiles for different values of  $v_o$

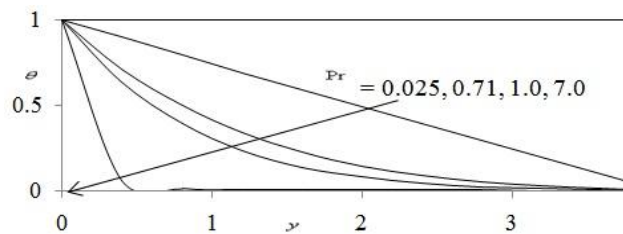


Figure 10. Temperature profiles for different values of  $Pr$

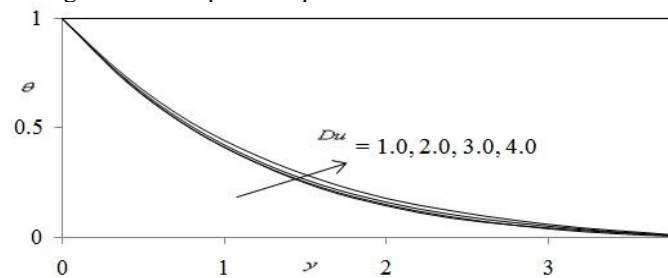


Figure 11. Temperature profiles for different values of  $Du$

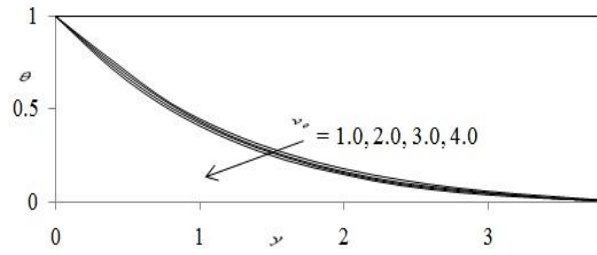


Figure 12. Temperature profiles for different values of  $\nu_0$

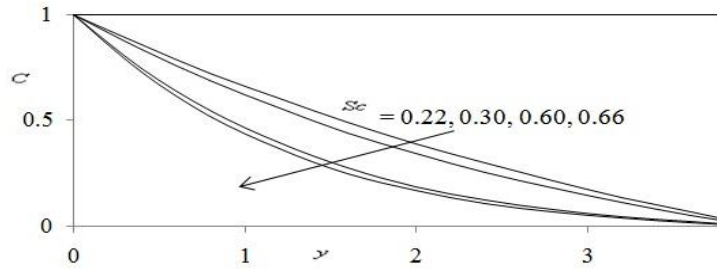


Figure 13. Concentration profiles for different values of  $Sc$

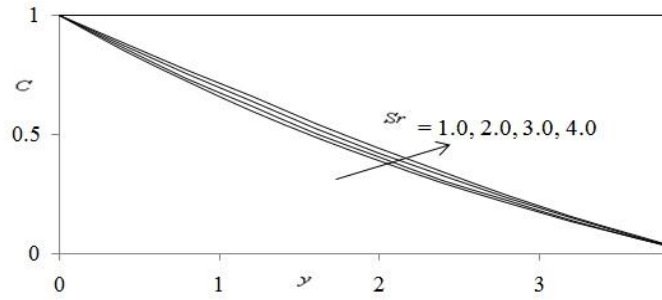


Figure 14. Concentration profiles for different values of  $Sr$

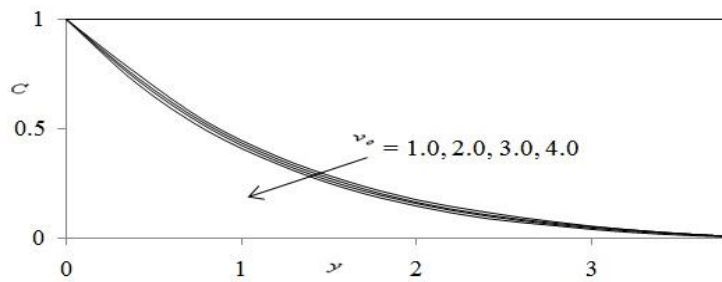


Figure 15. Concentration profiles for different values of  $\nu_0$