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Modelling and Forecasting Exchange Rates Volatility Using Arfima-Garch Model

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ABSTRACT

Volatility modeling and forecasting is a very important financial Time series research that has both theoretical and practical importance to the government, investors, academia, and private practitioners (modelers and forecasters), but it has received little attention. As a result, the current study serves as a link that bridges the existing gap in long-term memory stock in exchange rate volatility modeling and forecasting. The ARFIMA-GARCH model is used to analyze monthly exchange rate data for the Federal Republic of Nigeria and the Republic of Botswana from 2000 to 2019. According to the literature, the GARCH model is inadequate for modeling and forecasting financial and economic time series data. The study reveals that the exchange rates in Nigeria and Botswana are heteroscedastic and fractionally integrated processes. The results also revealed evidence of persistence, mean reverting tendency, and asymmetric effects in nature. The obtained results are invaluable policy documents for the two countries under study and to the well-being of the two countries' economies; it is also an invaluable policy document required by private practitioners, academia, and policymakers. It should be noted that linear models will be totally inadequate for resolving issues involving long memory.

Keywords: Exchange rate, GARCH, ARFIMA-GARCH, long memory, ADF and KPSS

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I. INTRODUCTION

The theory of econometrics and time series has not yet yielded the most effective method for forecasting exchange rates, despite the expenditure of effort and money. Cassel from 1923, Samuelson from 1964, Mundell from 1968, Allen and Kenen from 1980, Frankel and Mussa from 1985, MacDonald from 1999, Rogoff from 1999, Alba and Papell from 2007, Kim B.H. et al from 2009, Taylor from 2009, and Grossmann et al from 2010. (2009). Messe None of the forecast models can provide predictions for periods longer than a year, according to Rogoff (1983). In their studies of the likelihood of exchange rates, Alvarez and Alvarez-Diaz (2003, 2005, and 2007), Alvarez-Diaz (2008), Reitz and Taylor (2008), Anastakis and Mort (2009), Majhi et al. (2009), Bereau et al. (2010), and Bildirici et al. (2010) found conditional heteroskedasticity, leptokurtosis, and volatility convergence.The qualities mentioned here indicate a departure from normalcy. Akintunde et al. (2013) demonstrated the inadequacy of GARCH models for predicting exchange rates and proposed a hybrid of GARCH with Bilinear, GARCH with STAR models, and GARCH with ST models. With Ghana and South Africa as case studies, Alexander B. et al. (2017) used the ARFIMA approach to innovate GARCH and GJR-GARCH. The results of this study supported the presence of a mean return and the asymmetrical effects of economic shocks on the conditional means of the CPI inflation rates in the two nations under consideration. Cheung (2003) investigated the long-term memory of currency rates. Fung, Lai and Lo (2005), Peters (2006), Fisher et al. (2007), Barkoulas and Baum (2007c), and Chou and Shih (2007).Anderson et al. (2007) found long-memory in dollar/Deutschmark exchange rates using a GARCH model. For fractional differencing parameters, Barkoulas and Baum (2007a, 2007b) used spectral regression estimates and found that euro-currency returns had a long memory. Extended memory series feature continuous reliance across time-separated observations or nonperiodic long cycles. If exchange rates had a long memory, investors could anticipate price changes and, on average, make a profit. In nations with productive, well-managed pegs, random-walk dollar exchange rates should be applied. This happens as a result of daily mediation by central banks in the foreign exchange market to effectively support the peg, and as a result of their relatively small and steady trading volume. It is impossible to explain the idea of long memory without bringing up the autoregressive fractionally integrated moving average (ARFIMA) model. Long memory and ARFIMA will be introduced simultaneously, but because estimating ARFIMA models is challenging, it is always advisable to test for the presence of long memory first.2.0 MATHEMATICAL SPECIFICATION

2.1 LONG MEMORY AND VOLATILITY

A stationary process $(X_t)_{t\in T}$ is said to be a long memory process if there exists a real number $\alpha \in [0,1]$ and a constant $c_f > 0$ such that

$$f(w): \; c_f |w|^{-\alpha}, w \to 0 \tag{1}$$

$f(.)$ is a spectral density function. A class of process with the above property is called ARFIMA process, this process was initiated by Hosking JRM (1981). Mills (2007) generates ε_t autocorrelation recursively as

$$\rho_1 = \left\{\frac{d}{1-d}\right\},\; \rho_2 = \left\{\frac{d(1+d)}{(1-d)(2-d)}\right\} \text{ and}$$

$$\rho_k = \prod_{i=0}^{k}\left\{\frac{i-1+d}{i-d}\right\} = \frac{\Gamma(k+d)\Gamma(1-d)}{\Gamma(d)\Gamma(k-d+1)} : \quad \frac{\Gamma(\text{1-d})}{\Gamma(\text{d})}K^{2d-1} : \; mk^{2d-1}$$

2.2 ARFIMA MODEL

In general, X_t may be linked by zero mean ARMA process, $\Phi(B)X_t = \Theta(B)\varepsilon_t$ where ε_t is a white noise with variance, σ_ε^2, $\Phi(B) = 1-\phi_1 B - ... - \phi_p B^p$, $\Theta(B) = 1-\theta_1 B - ... - \theta_p B^p$ are the polynomials in B and $\phi_i = (i = 1,..., p)$ and $\theta_i = (i = 1,..., q)$ are the parameters of the autoregressive and moving average parameters respectively. Long memory can be found in the class of ARFIMA processes.

A time series $X_t = (X_1,..., X_t)$ follows an ARFIMA (p,d,q) process if:

$$\Phi(B)(1-B)^d X_t = \Theta(B)\varepsilon_t, \tag{2}$$

Where $\varepsilon_t \sim \text{iid}(0,\sigma^2)$, B is the backward shift operator, $\Phi(B) \equiv 1-\phi_1 B \_ ... \_ 1-\phi_p B^p$, $\Theta(B) \equiv 1+\theta_1 B + ... \_ 1-\theta_q B^q$, and $(1-B)^d$ is the fractional differencing operator defined by $(1-B)^d$ = $\sum_{k=0}^{\infty}\Gamma(k-d)B^k\Gamma(k+1)\Gamma(-d)$ with $\Gamma(.)$ being the gamma function. X_t is both stationary and invertible if the roots of $\Phi(B)$ and $\Theta(B)$ are outside the unit root circle and $d < |0.5|$ |. when $d = 0$, an ARFIMA process reduces to an ARMA process. Hosking (1981) showed that the auto-correlation, $\rho(\bullet)$, of an ARFIMA process satisfies $\rho(k) \propto k^{2d-1} as\ \text{k} \to \infty$. Sowell (1992) gives an algorithm for the computation of ACF of an ARFIMA process as,

$$\hat{\gamma}_k = \sigma_\varepsilon^2 \sum_{i=-q}^{q}\sum_{j=1}^{p}\psi_i\delta_j A(d, p+i-k, p_j).k = 0,...,T-1 \tag{3}$$

When $d \le 0$, the auto-correlations have a finite sum; that is, ARFIMA processes with $\text{d} \in (0,.5)$

Where $\psi_i = \sum_{s=|i|}^{q}\theta_s\theta_{s-|i|}$, $\quad \delta_j^{-1} = \rho\left\{\prod_{k=1}^{p}(1-\rho_k\rho_j)\prod_{m\neq j}^{p}(\rho_j-\rho_m)\right\}$; and

$$A(d,h,p) = \frac{\Gamma(1-2d)(d)_h}{[\Gamma(1-d)]^2(1-d)_h}$$

$$\left[\rho^{2p}F(d+h,1;1-d+h;\rho)\right] + \rho^{2p}F(d-h,1;1-d-h;p)-1 \;\;]$$

The function $F(.)$ is the hyper-geometric function such that

$$F(l;m;n;\rho)=\sum_{i=0}^{\infty}\frac{(I)_i (m)_i \rho^i}{(n)_i i!} \tag{4}$$

Where l,m and n are parameters of hyper-geometric function.

2.3 ARFIMA-GARCH MODEL

A stochastic process $(X_t)_{t\in Z}$ is an $ARFIMA(p,d,q)-GARCH(r,s)$ $p,q,r,s\in Nu(0)$ *and* $d\in R$ if it satisfies

$$\Phi(B)\nabla^d x_{(t)}=\Theta(B)\in_t \tag{5}$$

With $\in_t=\sigma_t z_t$

$$\sigma_t^2=\mu_0+\sum_{i=1}^{r}u_i\sigma_{\in}^2(t-1)+\sum_{j=1}^{s}v_i\sigma_{\in}^2(t-1)$$

(6)

$ARFIMA(p,d,0)-GARCH(1,1)$ is given by

$$f(t,w)=f_{SM}(t,w)\left|1-e^{-iw}\right|^{-2d} \quad -\pi\le w\le\pi$$

(7)

Where $f_{SM}(t,w)=\dfrac{\sigma_t^2}{\left|\phi\left(e^{-iw}\right)^{\alpha}\right|}$ is the spectral density of the corresponding shortrange correlations

$\sigma_{\in}^2(t)=1-\phi_1 z-\text{L}-\phi_p z^p$ *and* $\theta(z)=1-\theta_1 z-\text{L}-\theta_q z^q$ are polynomials such that $\phi(z)\neq 0$ *and* $\theta(z)\neq 0$

For $|z|\le 1$. B is a backward shift operator, ∇^d is the fractional difference operator that is defined as follows:

$$\nabla^d=(1-B)^d=1+\sum_{j=1}^{\infty}\frac{\Gamma(j-d)}{\Gamma(j+1)\Gamma(-d)}B^j,$$

$\Gamma(.)$ refers to gamma function $z_{(t)}$ is $i.i.d$ with mean zero and variance one, $\mu_0>0,$ $\mu_1,\text{L},\mu_r,v_1,\text{L},v_s\ge 0,$ $\sum_{i=1}^{r}u_i+\sum_{i=1}^{s}v_j<1$ *and* $\sigma_{\in}^2(t)$ refers to conditional volatility in equation 5, the parameter d dictates the long terms behavior of the series. Where p,q and the coefficient in $\Phi(B)$ *and* $\Theta(B)$ permits the modeling of short-range properties. Equation 6 elucidates the volatility process in a series such that the process emanating refers to its own lagged values and its squared residuals of the mean equation $-\frac{1}{2}<d<\frac{1}{2}$ when $ARFIMA-GARCH$ exhibits stationarity and invertible and $0<d<\frac{1}{2}$ when $ARFIMA-GARCH$ has a long memory and when the process is none stationarity and mean inverting it is

2.4 TEST FOR LONG MEMORY

2.4.1 RESCALED STATISTIC

This test was originated by Hurvich (1951) and was later modified by Mandelbrot and his co-authors. It refers to the range of partial sums of deviations from its mean, rescaled by its standard deviation. The rescaled statistic is defined as:

$$Q_T = \frac{1}{S_T}\left\{\max_{1\le k\le T}\sum_{j=1}^{k}\left(y_j-\bar{y}\right)-\min_{1\le k\le T}\sum_{j=1}^{k}\left(y_j-\bar{y}\right)\right\}$$

Where $\bar{y}=\frac{1}{T}\sum_{i=1}^{T}y_i$ and $S_T=\sqrt{\frac{1}{T}\sum_{i=1}^{T}\left(y_i-\bar{y}\right)^2}$ if y_t's are iid normal random variables, then

$$\frac{1}{\sqrt{T}}Q_T \Rightarrow V$$

Where $\Rightarrow$ denotes weak convergence and V is the range of a Brownian bridge in the unit interval. Lo (1991) pointed out that the R/S statistic is not potent to short range dependence. To permit for short range dependence in y_t, Lo (1991) modified the R/S statistic as follows:

$$\breve{Q}_T = \frac{1}{\hat{\sigma}_T(q)}\left\{\max_{1\le k\le T}\sum_{j=1}^{k}\left(y_j-\bar{y}\right)-\min_{1\le k\le T}\sum_{j=1}^{k}\left(y_j-\bar{y}\right)\right\}$$

2.4.2 GEWEKE AND PORTER-HUDAK (GPH)

Geweke and Porter-Hudak (1983) advanced a sturdier semi-nonparametric strategy to test for long memory. They opined that when the focus is on frequencies near 0, the parameter d can be estimated from the least squares regression

$$\ln\left(I\left(\omega_j\right)\right)=c-d\ln\left(4\sin^2\left(\omega_j/2\right)\right)+\eta_{j,}$$

Where $\omega_j = 2\pi j/T$ $\left(\text{j=1,..., T-1}\right), n=g\left(T\right)<T,$ and $I\left(\omega_j\right)$ is the periodogram of X at frequency ω_j defined by

$$I\left(\omega_j\right)=\frac{1}{2\pi T}\left|\sum_{i=1}^{T}e^{itw\left(X_t-\bar{X}\right)}\right|^2,$$

The sign of long memory is copious if the least squares estimate of $d\left(\hat{d}_{GPH}\right)$ is convincingly larger than 0.

2.4.3 LOCAL WHITTLE ESTIMATION

It is a semi-nonparametric procedure developed by Robinson (1995). It definition was related to parameter d and G as shown below

$$Q_m\left(G,d\right)=\frac{1}{m}\sum_{j=1}^{\infty}\left[\log\left(G\lambda_j^{-2d}\right)+\frac{\lambda_j^{2d}}{G}I_\alpha\left(\lambda_j\right)\right], \quad \text{(A)}$$

Where $m<n$. The local whittle estimate formed G and d by downplaying $Q_m\left(G,d\right)$ in a way that

$$\left(\breve{G},\breve{d}\right)=\underset{G\in(0,\infty),d\in\left|\Delta_1,\Delta_2\right|}{\arg\min}Q_m\left(G,d\right), \quad \text{(B)}$$

Where Δ_1 and Δ_2 are numbers such that $-\frac{1}{2}<\Delta_1<\Delta_2<\infty.$. Using Robinson (1995), by focusing (A) in (B), gives $\hat{d}=\underset{d\in\left|\Delta_1,\Delta_2\right|}{\arg\min}R\left(d\right),$

Where $R\left(d\right)=\log\breve{G}\left(d\right)-2d\frac{1}{m}\sum_{1}^{m}\log\lambda_j$ $\breve{G}\left(d\right)=\frac{1}{m}\sum_{1}^{m}\lambda_j^{2d}I_x\left(\lambda_j\right).$

3 DATA ANALYSIS AND INTERPRETATION OF RESULTS

Unit Root Test (level)

Augmented-Dickey Fuller test was used as a test of unit root at 5% and 10% for both the level and first difference. At level the series was chaotic, volatile and unstable, but the series was stable at first difference. The long memory property of the series was asserted.

Table 1: Unit Root Test (level)

Series	ADF-Test statistic	Critical value 5%	10%	Mackinnon prob ●
Naira	0.433965	-2.5702	-2.8678	0.9842
Pula	0.2013	-2.5702	-2.8678	0.9725

Source: extracts from E-view12 outputs

Table 2: Unit Root Test (First difference)

Series	ADF-Test statistic	Critical value 5%	10%	Mackinnon prob ●
Naira	-20.013751	-2.5702	-2.8678	0.0000
Pula	-21.6683	-2.5702	-2.8678	0.0000

Source: extracts from E-view12 outputs

PRE-ESTIMATION TEST FOR ARCH EFFECT IN THE NIGERIAN AND BOTSWANA EXCHANGE RATE DATA.

TABLE 3: HETEROSKEADSTICITY TEST: ARCH EFFECT

	Nigeria	Botswana		Probability Nigeria	P-value Botswana
F-Statistic	17.3218	10.2310	1.2111	1.0231	0.0000
Observed R-squared	14.2312	8.9991	1.1111	1.0027	0.0000

Source: extracts from E-view12 outputs

Table 3 above reveals the results of the test for ARCH effect of the series under study. The F-Statistic value obtained lead to the non-acceptance of the null hypothesis for lack of ARCH effect and the acceptance of the alternative hypothesis for its presence in the series. It should be emphasized that the use of ARFIMA-GARCH is only possible if and only if there exist conditional heteroskedasticity in the series under study otherwise it reduces to ARFIMA model. From table 3 above there exist conditional heteroskedasticity because of the acceptance of alternative hypothesis and as a result we can estimate ARFIMA-GARCH model for the study.

TABLE 4: ARFIMA (3,1,1)-GARCH (1,1) RESULTS FOR NAIRA

Variable	*Estimated coefficient*	*standard error*	$t-test$	$p-value$
d	0.2626	1.6249	2.2226	0.0024
w	0.6883	0.3847	1.2600	0.2124
α	0.3412	0.3691	0.7182	0.3826
β	0.0763	0.3035	0.2113	0.4718

Source: extracts from E-view12 outputs

TABLE 5: ARFIMA (3,1,1)-GARCH (1,1) RESULTS FOR PULA

Variable	*Estimated coefficient*	*standard error*	$t-test$	$p-value$
d	0.2728	0.3411	1.2151	0.0021
w	0.4132	0.2899	0.0821	0.3086
α	0.2276	0.2391	0.2491	0.2142
β	0.3121	0.7821	0.6211	0.2613

Source: extracts from E-view12 outputs

Tables 4 and 5 show the results of ARFIMA-GARCH estimates for exchange rates of Nigeria (Naira) and Botswana (Pula) as extracted from the analysis. d value (fractional integrated parameter) obtained is significant, indicating that the exchange rates is independent and identically distributed process. d value is also very close to zero indicating the volatility of the series under study which confirms the results obtained for Augmented Dickey Fuller test for long memory and for unit root test.

LONG MEMORY PARAMETERSESTIMATE OF NIGERIAN NAIRA AND BOTSWANA PULA.

TABLE 6: LONG MEMORY PARAMETERS ESTIMATE

Test	Nigeria exchange rate	Botswana exchange rate
Fractional integration parameter		
Local Whittle estimator (LWE)	0.28 $(0.000)^{\bullet}$	0.56 $(0.000)^{\bullet}$
Geweke and Potter Hudak (GWH)	0.36 $(0.000)^{\bullet}$	0.52 $(0.000)^{\bullet}$
Rescaled Statistic (RS)	0.51 $(0.000)^{\bullet}$	0.64 $(0.000)^{\bullet}$

Source: extracts from E-view12 outputs

It should be noted that in the two cases the null hypothesis of no long memory is not accepted at 5% significance level.

4.0 SUMMARY OF MAJOR FINDINGS AND CONCLUSION

Augmented-Dickey Fuller testwas used to test for the presence of unit root test at 5% and 10% for the level and first difference. At level the series was chaotic, volatile and unstable, but the series was stable at first difference. above reveals the results of the test for ARCH effect of the series under study. The heteroscedsticity test conducted reveals that F-Statistic value obtained lead to the non-acceptance of the null hypothesis for lack of ARCH effect and the acceptance of the alternative hypothesis for its presence in the series. there exist conditional heteroskedasticity because of the acceptance of alternative hypothesis and as a result ARFIMA-GARCH model for the study was estimated. The results of ARFIMA-GARCH estimates for exchange rates of Nigeria (Naira) and Botswana (Pula) as extracted from the analysis. d value obtained is significant, indicating that the exchange rates is independent and identically distributed process. d value is also very close to zero indicating the volatility of the series under study which confirms the results obtained for Augmented Dickey Fuller test for long memory and for unit root test.

In conclusion the existence of long-memory in exchange rates shows significant relationships between observations. Applying advanced statistical models like ARFIMA-GARCH model for forecasting will not only serve the interest of forecasters, academia and investors, but also assist financial timeseries cum econometricians to be able to assess the empirical behavior of financial data. Performance in the foreign exchange markets. The ARFIMA-GARCH process remains the choice as it is flexible in modeling the short- and long-term forecast properties of the exchange rates.

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