Research Inventy: International Journal of Engineering And Science
Vol.10, Issue 2 (Feb 2020), PP 12-18
Issn (e): 2278-4721, Issn (p):2319-6483, www.researchinventy.com

Irreversibility in Quantum Computing and the Relation to Space-Time Crystals

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Abstract:*Experimentally observed space-time crystals are described here as sequences of quantum parallel computing processes which have the appearance of reversible computing in each sequence. We present here a quantum system consisting of an entangled atomic chain modeled in a deterministic cellular automaton system to show that the periodic reappearance of an initial computational state does not imply reversibility in the quantum computing system. The origin of this irreversibility is in the waste of half the time steps in generating the observable discrete space-time crystals. This is due to the simultaneous existence of both clockwise and counterclockwise cyclic CA transition rules for the computational states in generating the quantum computing results. Thus we predict that any attempt to observe those individual computational CA sequences will be only half successful even though the value of the periodicity itself can be correctly established. The results shown here are very general for any initial CA configurations composed by those four cyclic computational states, which are linked by a unitary transformation to the four orthogonal electronic states of the atom in each cell to form a string of entangled atoms.*

Keywords:*Space-Time Crystals, Quantum Computing, Quantum Information*

Date Of Submission: 20-05-2020 Date Of Acceptance: 05-06-2020

I. INTRODUCTION

Parallel computing, classical or quantum, deals with spatial transformation rules. It is sufficient to deal with just four computational states defined as: S1 = (0,0), S2 = (0,1), S3 = (1,0),and S4 = (1,1). Each state is then associated with a specific black-and-white spatial pattern. For example, any addition operation can be achieved with the manipulations of two long bit-strings that are composed by those four computational states (1) The quantum processor needed to perform such manipulations are manmade and the computing architecture is necessarily in cellular automata, which utilizes a two-bit cell, if the interconnections between the processors are to be at the minimum. The evolution of two long bit-strings that forms the initial computational states reflects the capability of the quantum processor resided in each CA cell to achieve the desirable results, such as obtaining the correct final computational state of the addition operation resided in each cell. The computational states are clearly the observables and are all real quantities. This is certainly the case for the general-purpose quantum computing (1). Now if the man-made processor is replaced by an atom to perform some special-purpose quantum computing, the four computational states must remain in place to be used as the observables. But the observables now reflect the capability or the inner working of the electron wave functions of the entangled atoms. In other words, the evolution of the computational states as the observables are the results of the interferences or the constraints of the phases at each electron energy among the entangled atoms. If the four computational states are sufficient for general-purpose computing, then they are also enough for the special-purpose computing performed by the entangled atoms. Consequently, the corresponding atom resided in each cell needs to have just four electron energies, E_1, E_2, E_3 and E_4, and the four corresponding orthogonal states, $\overrightarrow{S_1}$, $\overrightarrow{S_2}$, $\overrightarrow{S_3}$, and $\overrightarrow{S_4}$. The relation between the four computational states and the four orthogonal states in each cell is linked by the unitary transformation as dictated by the quantum mechanics. But the entire CA evolution is not related by a unitary transformation from one iteration to the next as we would like to emphasize here from the start. The net result is that the computational states now exhibited in the form of discrete time crystals and with birth-and-death of a space-time block elsewhere (1).

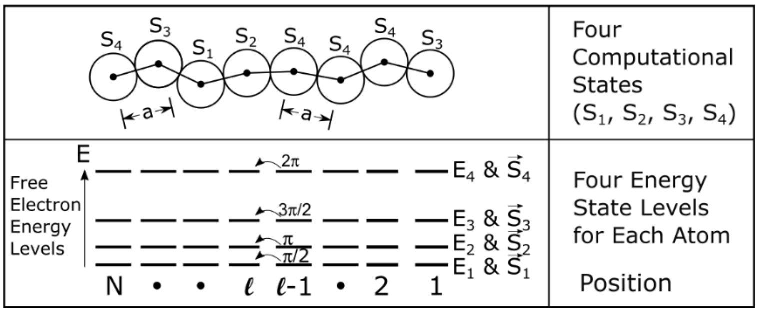
The periodical appearance of a space-time block in CA clearly shows a deterministic CA evolution sequence in the sense that after the first appearance of the space-time block, all the subsequent structure of the CA evolution is determined and predictable in four different space-time block forms (2). This is the characteristic of a special class of CA evolutions having an equal probability of transition among the four computational states. The deterministic nature of the CA evolution is clearly also a class of reversible CA exhibited by a structured Sierpinski triangle (2).

But the question is: in view of the reversible CA having periodical appearances of four different basic space-time blocks, can the actual quantum computing itself reversible as an observable result at all? We show the generation's compustational states are only half-observable. The other half are in superposed states and therefore half of the time steps are wasted as far as computation is concerned. This is the center of our investigation. Having a reversible CA as indicated by the predictable computational states does not guarantee the reversibility of quantum computing because there are two sequences of concurrently superposed computational states in the quantum nature, one is clockwise cyclic and the other counterclockwise cyclic (the conjugate) in the observables. That means the phase relation of the electrons among the entangled atoms are dual-valued in the computing processes. The phase values of transferring the four orthogonal states for an electron from one cell to the next are$\pm\pi/2, \pm\pi, \pm 3\pi/2, and\ \pm 2\pi$. The $\pm$ signs for the two half-integer phases have no effect on the results of the computational states. But the two integer phases for the clockwise and counterclockwise are superposed in two different computational states and become un-observable. This implies half of all the time steps are wasted by the design of the quantum nature through its overcapacity in time steps for computational purpose. That means each of the clockwise and counterclockwise computational states can only be observable in half of the time steps. In the other time steps, two different outcomes are superposed and cannot be separated, even though there are two concurrent CAs that are reversible individually. In other words, the quantum nature provides us twice the amount more than we need to observe in the form of computing. This is our main result and is very general for any initial computational-state configurations.

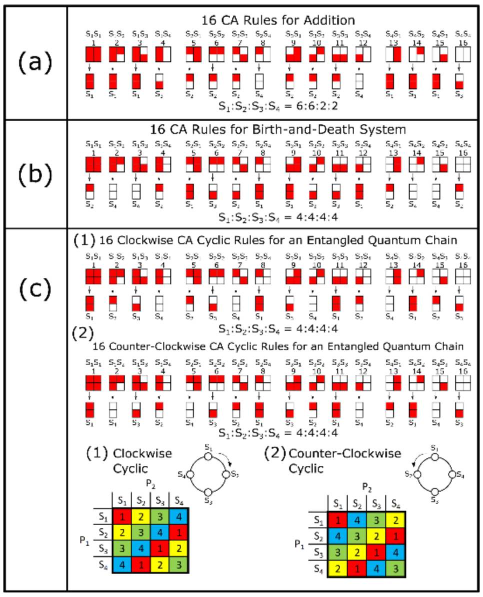
Thermodynamic irreversibility deals with waste of energy that is not recoverable. Computing irreversibility correspondingly deals with data that is not recoverable, data which costs energy to both generate and erase. For example, in an addition operation if neither of the operands are saved then the operation is irreversible. Here, saving the data is a form of waste of space. Parallel computing, however, involves not only space, but also the instruction capability and the time steps needed to generate the data. Therefore, the concept of computing irreversibility must also be extended to include both the instruction capability and the time steps that are built-in in the system in generating the computation. This is manifest in the overcapacity of the instructions provided from a quantum processor which cannot be removed or separated from the quantum processor, or from the overcapacity of the pertinent time-steps, which is the center of our investigation here. Birth-and-death of space-time blocks shows an example of waste of instruction capability (2). Here we present the third kind of computing irreversibility: waste of time steps designed by the quantum nature for our observation. This is in addition to the two kinds of computing irreversibility shown earlier (1, 2). Here we would like to make clear that irreversibility is not a property of the CA, which is itself reversible and hence why we do not discuss irreversibility in the CA shown here, but rather the irreversibility is in the actual computing itself.

Recently, the demonstrations of space-time crystals from theory to experimental verification (3-37) implies a reversibility of computing in such a quantum system in crypto equilibrium, since any initial computational state from a chain of entangled atoms will repeat itself under periodic external perturbation, a Floquet system (22).Each external perturbation, such as a laser pulsing on each atom to cause a transition between the energy levels within the atom, requires a minimum waste of energy at the amount of kTln2, as shown by Landauer (30). That is the thermal energy times the amount of the information content of changing one bit, the entropy. In addition, thermodynamic irreversibility lies the appearance of computational reversibility under periodic perturbation of the quantum system.

The question here is whether such quantum computing processes are reversible under the appearance of the space-time crystal generations. The signature of space-time crystals from such periodic laser pulses is that the size of the crystals always lies between Nx2N and Nx4N, regardless of the initial computational states.Here N is the number of atoms of the entangled atomic chain.The signature of a space-time crystal generation is the value of sub-harmonic limited to between $\frac{1}{2}$and $\frac{1}{4}$,independent of the period of the perturbation.The evolution of an entangled atomic chain under such a periodic perturbations is itself a quantum parallel computing process, because the computational state of every atom is altered at each iteration simultaneously.



A finite non-Euclidean space-time crystal is illustrated as an entangled atomic chain of size N and spacing a, shown here with N=8 and an initial computational state of $S_4S_3S_1S_2S_4S_4S_4S_3$. Each atom is associated with four free-electron energy levels and four corresponding orthogonal states. Four different phase gains corresponding to the four energy levels of the atom are incurred when an orthogonal state is transported from l-1 to 1 as indicated. However, during the transport there are two possible phase transformations possible depending on the transition rule, one clockwise and the other counter-clockwise. The results of from each transition rule are mutually conjugate.

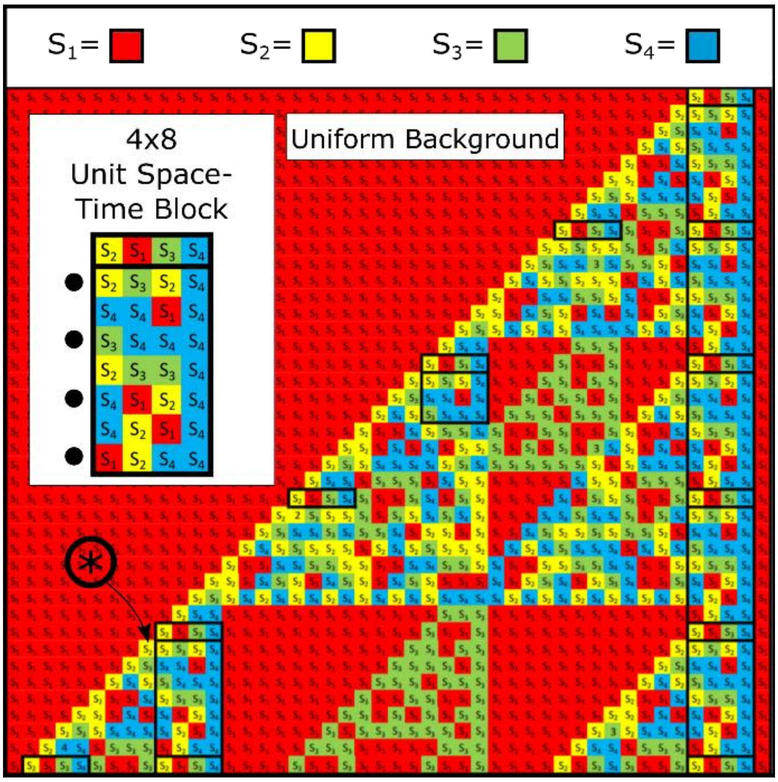


Three different sets of 16 CA spatial transformation rules for computational states: Set (a) is the rule for an addition operation of two n-bit integers. Here, the computational states must be identified with the operand pairs as $S_1 = \binom{0}{0}, S_2 = \binom{0}{1}, S_3 = \binom{1}{0}, and\ S_4 = \binom{1}{1}$. Note that the S_1 and S_2 states have three times the survival probability in each CA iteration compared to the S_3 and S_4 states, and the CA chain also must be Euclidean. A finite rectangular space-time block is generated in the addition operation. Set (b) demonstrated the birth-and-death of space-tine blocks of size NxN in an infinite helical chain. Both (a) and (b) can utilize the same quantum processor but with a different interconnection scheme. Note here that all four computational states have an equal survival probability after each iterations. Set (c) is for a quantum CA system where each atom is the quantum processor as depicted in Fig. 1. The corresponding four computational states in this system must have an equal survival probability at each transition, and in addition be cyclic as shown. There are two cyclic sequences that are equally probable in nature, one clockwise (c_1) and the other counter-clockwise (c_2).

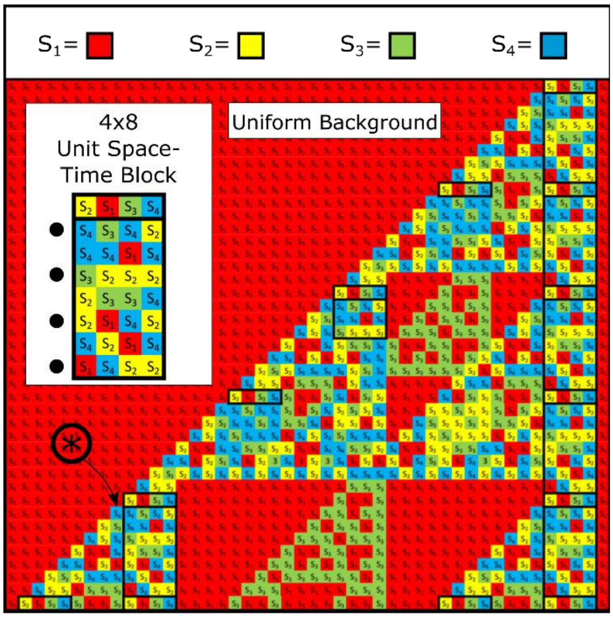
The generation of a time crystal can then be modeled within a framework of cellular automata (CA). This is recently described (1) using a model quantum system where each atom in the chain has 4 free-electron energy levels, $E_m = \frac{h^2}{32m_\varepsilon}\left(\frac{m}{a}\right)^2$, corresponding to four orthogonal states $\overrightarrow{S_1}, \overrightarrow{S_2}, \overrightarrow{S_3}$, and $\overrightarrow{S_4}$. Here m = 1,2,3,4 and a is the spacing between atoms and m_ε, the electron mass (Fig. 1). The evolution of the initial computational state of each atom is constrained by the relation to its neighboring atoms through an entanglement environment. The four computational states, S_1, S_2, S_3, and S_4, are each a linear combination of the four orthogonal momentum-vector states through the unitary transformation. The Hamiltonian of theentangled atomic chain can also be conveniently built-in through the use of a cellular automata (CA) architecture without the need to construct it as we have shown earlier (1). When CA is applied to a quantum system, an operator must operate on a state at the same location due to the nature of quantum mechanics. In CA, however, an operator acts on its neighboring state as indicated by the transition rules, while simultaneously being a state in itself and depends on the neighboring relations. Together, that means a quantum operator (a computational state as an operator) must be transported to its neighboring site first before acting on that neighboring state. Since a computational state is a linear combination of the four orthogonal states of the atom as defined above, each of the orthogonal states will acquire four different phases when they are transported to the neighboring site. This results in the relation$\overrightarrow{S_m S_m} = \delta_{m,n} e^{im\pi/2}\overrightarrow{S_m}$where m,n = 1,2,3,4. Note that the quantum operator$\overrightarrow{S_m}(l-1)$, is located at position l−1 while the operand, $\overrightarrow{S_n}$, is located at its neighbor of l. Thus, there are four different phases, π/2, π, 3π/2, and 2π, acquired for the four different energies when the four orthogonal states of the atom are transported to its neighboring site.

It is then important to note that if those four phase relations are to be maintained under the transportation, then the corresponding CA transition rules for the four computational states, through the unitary transformation, must be of equal probability and cyclic as was shown earlier (1-2). This establishes that quantum CA system with its Hamiltonian automatically built-in and without using the Euclidean addition rules (1). Those relations are shown in Fig. 1 in an example, where the initial configuration of N=8$S_4S_3S_1S_2S_4S_4S_4S_3$.The equal probability, cyclic transition rules are shown in Fig. 2 (c_1, c_2).

The advantage of bringing a quantum system into the CA computing architecture is that there is now no need for a Hamiltonian formulation of an entire system when the cyclic computational rules are imposed.If the proper Hamiltonian needed is shown to be decomposed into several additive components, then the cyclic rules used here will be inconsistent with the addition rules of the Hamiltonian.In other words, the additive terms in the Hamiltonian force one to use the addition-rule-compatible CA operations shown in Fig.2 (a), and the cyclic rules used here, Fig. 2 (c_1, c_2)are inherently incompatible with addition-rule-based CA.That is the reason why a larger and complex quantum system is more efficient or more convenient to be built up by smaller quantum components directly, such as an entangled atomic chain.This is analogous to a multiplication operation being built by many CA chains of addition operations.



The space-time crystal generated from the clockwise cyclic rules of size Nx2N is shown on the right columns in black blocks. The initial configuration is $S_1S_2S_3S_4$. For an infinite quantum chain, birth-and-death of space-time crystals can be generated as shown by the block marked '*' which is identical to the original 4x8 unit space-time block.This is an example of teleportation in the quantum system.



The corresponding space-time crystal generated from the counter-clockwise cyclic rules is shown with the same initial configuration as in Fig. 3. This crystal differs from Fig. 3 on every other row as indicated by the black dot. Half of the data or the computing generated from the two cyclic rules is wasted because they are not useful to the generation of the space-time crystal in nature.

In a finite CA with an infinite confinement wall, as in those experimental settings (22), the evolution of a computational state of each atom in CA will be governed by the phase changes incurred from the neighboring relations, but each site with various periodicities. When all periods converge in all cells, it establishes the period of the space-time crystal generated. This is shown for an arbitrary initial computational states using the c1 rule from Fig. 2 (Fig. 3). The space-time crystal shown here is of the size Nx2N, if N is an integer of 4. Generally the size is limited between Nx2N and Nx4N and the space-time crystals themselves reflect the associated quantum parallel computing processes, just like the addition of two bit-strings,using man-made quantum processors. The evolution of an arbitrary initial computational-state having undergone the CA iterations is a quantum parallel computing process on the initial state as performed by an entangled atomic chain where each atom is a quantum processor with four instructions. This is much like an array of interconnected man-made quantum processors in CA to perform an addition operation of two long bit-strings. This part has been demonstrated earlier by the use of a man-made quantum processor with the capacity for four instructions and the storage of two data bits (38–41) (also US Patent #8,525,544) using Aharonov-Bohm-based quantum networks (40).

II. DISCUSSION

In the situation of an addition operation, computing is irreversible if one of the two operands is not saved along with the result. This irreversibility creates a condition where too much data must be saved over the course of a calculation in order to retain reversibility. The origin of this irreversibilityshownintheadditionoperationis also found in the 16 Corresponding Author: transitionrules, where two of the computational states (S1 and S2) have three times the probability to survive than the other two states (S_3 and S_4) at each iteration (Fig.2(a)). However,in quantum computing systembyanentangledatomicchain, the(cyclic) transitionrulesusedprovideanequalsurvival probability among the four computational states as shown in Fig 2 (c1,c2). The existence of space time crystals raises the important question: whether or not such a quantum computing process is actually reversible. Irreversibility in computing generally means a waste of space such that some data is to be destroyed or reduced to a smaller data set in order not to waste the data space. This applies addition operations and beyond in general-purpose computing.

The concept of irreversibility in parallel computing must be expanded to include two other forms of waste: waste of instruction capabilities and waste of time steps in the computing process.In the birth-and-death of space-time blocks demonstrated using the same quantum processor used in the addition operation but with different interconnections, we showed that only half of the instruction capabilities are utilized in the steady state, and only the first two steps require the full instruction capacity.Thus a waste of instruction capabilities is a form of irreversibility (1, 2), even though the CA is reversible in the form of a structured Sierpinski triangle (2, 41).In quantum systems that result in space-time crystals, the irreversibility in computing means un-avoidably there are data un-necessarily or un-usefully generated in part of the time steps that are not readable due to superposition of the two sets of data simultaneously present in every other time steps.In other words, a waste of time for producing readable data steps is another form of irreversibility.In the example of the Nx2N crystal in Fig. 3 using the CA rule from Fig. 2 (c_1), one can argue that half of the time steps are wasted because there exists another set of cyclic rules, Fig. 2 (c_2) and Fig. 4, which occurs with an equal probability in nature. These two rules are conjugate to each other from + and – signs of allowed phase changes of the orthogonal states. Hence we denote the ”clockwise” and counterclockwise” sequences of the computational states. This results in a generation where every other row compared between two crystals with the same seed generated under these two conjugate rules is identical, and the intermediate rows differ, and the computational states S_2 and S_4 are swapped between them.For this second cyclic rule, the momentum-vector transition is given by $\overrightarrow{S_m S_m} = \delta_{m,n} e^{im\pi/2} \overrightarrow{S_m}^*$.This does not affect the transition for states $\overrightarrow{S_2 S_2}$and $\overrightarrow{S_4 S_4}$, but causes the transitions for states $\overrightarrow{S_1 S_1}$and $\overrightarrow{S_3 S_3}$ to become conjugate. Since clockwise and counterclockwise systems are equally probable in quantum mechanics, the useful data generated in the Nx2N space-time block is only found in the time steps when both sequences generated identical data, as in the intermediary steps, data from the clockwise and counter-clockwise generations are superimposed producing a wasted time step in the computation. This creates the equivalent of an NxN block of useful or readable information, even though the periodicity is 2N.The intermediate steps form a waste of the time steps, but nevertheless must stay in the crystal generation.This example shows how reappearance of an initial seed does not necessarily imply reversibility in computing, as both crystal generations under the same initial configuration but with different rules, show a reappearance of that same seed at the same time intervals, and are also identical in one half of the time steps.

III. CONCLUSION

The irreversibility in computing is connected with how data generated are necessarily removed and hence not recoverable.The necessity of saving the data space applies to the situation of addition operations and beyond in general-purpose computing.The space-time blocks generated in an addition operation are generally rectangular, and the one-dimensional CA has to be in Euclidean space because addition rules are associated with straight lengths.This concept is first expanded to include how data are generated by a man-made quantum processor other than the spatial transformations for addition rules.If half of the instruction capabilities of the processor are not used (that means, out of the 16 CA rules, only eight are used in the steady state as in the rule set shown in Fig. 2 (b)), a full capacity has to be maintained with the processor. Nevertheless the waste of the capability is a form of irreversibility.This applies to many CA systems with an equal probability of transitions among all the computational states.The space-time blocks generated are of size NxN, and the space utilized is helical, such as in DNA, because the interconnections between the processors are different from the interconnections for addition operations where Euclidean space is required.An Euclidean space is the singular requirement that is established for the addition operation and hence this is the only singular set of 16 CA transition rules that produce the addition rule as shown in Fig. 2 (a).Any deviation from this particular set of rules, which are for the rest of 4^{16} sets, will bring us to utilize Non-Euclidean space.The helical chain is also an example for DNA with CGTA-type of four computational states, and birth-and-death of space-time blocks is generated if the space is infinite.

In the third example here of an entangled chain of atoms, the quantum processor is the atom itself and not man-made.The orthogonality of 4 atomic states in each atom of the model quantum system implies the four computational states for the CA transition rules are not only of equal probability but also have cyclic transitions.The space-time crystals generated are in Nx2N blocks where the size of the initial atomic chain is N and N is a power of 2.In this case, the computing irreversibility is caused by the waste of half of the time steps used to generate the space-time crystals.The reason is that there are two probable cyclic sequences in the quantum nature, one is clock-wise and the other counter-clockwise, or equivalently the + and – signs of electron phases assigned for transferring an orthogonality states of the electron to its neighbor.They generate the space-time blocks which are identical in every other step. Since both cyclic rules are equally probable in quantum mechanics and hence the unreadable or the superposed data are generated half of the time without further processing. The waste of half the time steps in the generation of these space-time crystals is the cause of this quantum computing irreversibility.That means in the quantum nature, a single CA chain can run two concurrent computational-state transition rules simultaneously through the dual-valued phase transfer of the electron from cell to cell, but only produces useful computational data on every-other row, producing a waste of time steps and hence computing irreversibility.

Conflict of interest
There is no conflict to disclose.

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C. H. Wu, et. al. "Irreversibility in Quantum Computing and the Relation to Space-Time Crystals." International Journal of Engineering And Science, vol. 10, no. 02, 2020, pp. 12-18.